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Review Article

Oikoumene and the Dynamic Casimir Effect

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Abstract

This paper re-examines the dynamic Casimir effect as a possible mechanism for propulsion. Previous investigations assumed mechanical motion of a mirror to generate thrust. In this case, because of the finite strength of materials and the high frequencies necessary, the amplitudes of motion must be restricted to the nanometer range. To permit larger amplitudes, an epitaxial stack of transparent semiconductor laminae is proposed. Voltage is rapidly switched to successive lamina, creating continuous, large amplitude motion of a reflective surface without mechanical contrivances. The paper provides relativistic results for large amplitude motion. With centimeter-level magnitudes, propulsive forces are raised to significant levels.

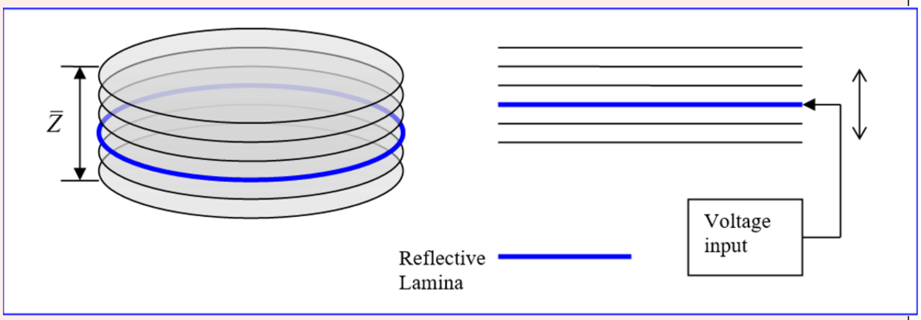
Introduction

More than three-quarters of a century ago [1,2] explained the retarded van der Waals force in terms of the zero-point energy of a quantized field. Both the static and dynamic Casimir effects are discussed in several reviews [3-7]. This work is concerned with the dynamic Casimir effect, which involves the interaction between moving mirrors and the ground state ("vacuum state") of the electromagnetic field. In particular, following Maclay and Forward, [8], the present analysis is motivated by the possibility of a propulsive mechanism.

When estimating the magnitude of the force that could be generated, Maclay and Forward assumed that the amplitude of high frequency motion of an actual mirror need be in the nanometer range due to the finite strength of materials. This restriction limits the possible propulsive force to very small values. However, this author observes that motion of a single reflective surface is not essential: that the Casimir effect is due to the motion of the boundary conditions constraining the free field in its ground state. The advent of amorphous oxide, transparent semiconductors used for thin film applications [9-14] suggests the possibility of achieving large motions of reflective surfaces without mechanically moving parts. The proposed epitaxial assembly of semiconductor laminae, is illustrated in Figure 1. Without the application of voltage, each lamina is a partially transparent dielectric; but when supplied by voltage it becomes a reflecting conductor serving as a mirror. Voltage inputs can be switched among the laminae at high speed, effectively moving the mirror at high velocities and accelerations without the use of moving parts. Thus, motions of the reflective surface that have both high frequencies and large amplitudes can be produced. In a treatment of the pressure on moving mirrors due to the Casimir effect, Neto and his colleagues, [7], took a perturbative approach consistent with the assumption that the mirror motion be constrained to very small amplitudes. The objectives of this paper are to extend the analysis to large motions and the epitaxial approach described above; to obtain explicit expressions for the forces produced by a particular trajectory of motion; and to estimate the numerical values of these forces.

It is assumed that within certain wavelength bands, the reflectivity of each lamina can be set within a continuous range from completely reflective to completely transparent. The laminae are also characterized by a finite response time. These features can be combined so that when the laminae are sufficiently closely spaced, and their energizing processes are properly phased, multi-laminae propagating wave of reflectivity can be created that sustains the properties of a continuously moving mirror (see Appendix A). In the following, the multi-laminae phasing wave is treated as a single, perfectly reflecting surface.

Figure 1: Epitaxial stack of semiconductor lamina.

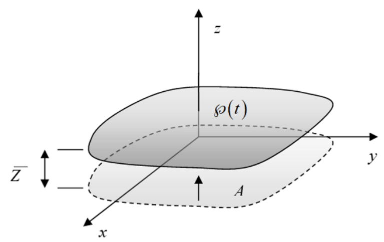


Fundamental Development

Define a coordinate system, (x, y, z), with unit orthogonal basis vectors $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ Consider the case in which surface $\wp(\tau)$ is a section of a plane having area A and parallel to the x-y plane, as illustrated in Figure 2. Its motion is along the z axis with z-coordinate $q(t)$where $q(t) \in [0, \bar{Z}]$ Before the reflectivity is "turned on" at $t = 0$ the field is in the vacuum state. Also, the surface starts a cycle of motion coinciding with the x-y plane, so that $q(t=0) = 0$ It is assumed that $\bar{Z} \ll \sqrt{A}$ so that one may treat the conductive surface and the field it produces without accounting for edge effects. In the following the x-axis is defined to be perpendicular to z and in the plane formed by z and the direction of propagation of the plane wave associated with a particular vacuum state mode.

For simplicity in this initial development, the surface is presumed to be either perfectly reflecting or perfectly transparent, depending only upon the wave number. This is modelled

Figure 2: Geometry of the motion of the conductive surface.



as a scalar function, $R(k) = 1$ (reflective), $R(k) = 0$. As a minimum, a model of $R(k)$ should capture the fact that any conductive material is transparent to radiation that has frequencies above the plasma frequency, . Following the Drude-Sommerfeld model [15-17], the upper limit of the wavenumber might be some fraction of:

$$\bar{k} = \omega_p / c$$
$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m^*}} \quad \left(= 8980\sqrt{n_e}\ Hz\right) \quad (1.a,b)$$

where m^* is the effective mass of the charge carriers, e is the elementary charge, and n_e is the volumetric number density of the charge carriers. ω_p may be typically $\approx 10^{14}\ Hz$ and the value for metals can be a hundred or even a thousand-fold larger. The simplest model has the form:

$$R(k) = \begin{cases} 1, & k < \bar{k} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

(2) is essentially a formal regularization since the details of the dielectric function of the materials, the effects of absorption, and the semiconductor design and parameters are ignored. The reflective properties are conceived to be homogeneous and isotropic.

To begin the analysis, the notational conventions of [18] are followed. Also, the continuous Fock space approach to quantizing the electromagnetic field [19] is adopted. The electric field operator in empty space and in the absence of boundaries is given by:

$$\hat{\mathbf{E}}(\mathbf{r},t) = \frac{i}{(2\pi)^2} \sum_{s=1}^{2} \int \sqrt{\frac{hck}{2\varepsilon_0}} \left[\hat{a}(\mathbf{k},s) \boldsymbol{\varepsilon}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - h.c. \right] d^3k \quad (3)$$

where ε_0 is the vacuum dielectric constant. Quantities in bold type are 3-vectors, and a carrot over the symbol indicates a quantum operator. "h.c." stands for "Hermitian conjugate". is the continuous wave number vector and ω is the angular frequency, where $\omega(\mathbf{k}) = ck,\ k = \|\mathbf{k}\|$. $\boldsymbol{\varepsilon}(\mathbf{k},s), s = 1,2$ are the polarization vectors obeying the orthonormality requirements:

$$\mathbf{k}\cdot\boldsymbol{\varepsilon}(\mathbf{k},s) = 0, \quad (s = 1,2)$$
$$\boldsymbol{\varepsilon}^*(\mathbf{k},s)\cdot\boldsymbol{\varepsilon}(\mathbf{k},s') = \delta_{ss'} \quad (s,s' = 1,2) \quad (4.a\text{-}c)$$
$$\boldsymbol{\varepsilon}(\mathbf{k},1) \times \boldsymbol{\varepsilon}(\mathbf{k},2) = \mathbf{k}/k \triangleq \boldsymbol{\kappa}$$

The terms $\hat{a}(\mathbf{k},s)$, and $\hat{a}^\dagger(\mathbf{k},s)$ are the annihilation and creation operators for field modes of wave vector , and polarization . These obey the commutation relations:

$$\left[\hat{a}(\mathbf{k},s), \hat{a}^\dagger(\mathbf{k}',s')\right] = \delta^3(\mathbf{k}-\mathbf{k}')\delta_{ss'}$$
$$\left[\hat{a}(\mathbf{k},s), \hat{a}(\mathbf{k}',s')\right] = 0 \quad (5.a\text{-}c)$$
$$\left[\hat{a}^\dagger(\mathbf{k},s), \hat{a}^\dagger(\mathbf{k}',s')\right] = 0$$

Equation (3) describes the free electromagnetic field, which is taken to be the condition of the field at the initial instant, t=0 . Note that the quantized fields are coupled by the same Maxwell Equations as the classical fields from which they came, i.e.:

$$\nabla \times \hat{\mathbf{E}}(\mathbf{r},t) = -\frac{\partial}{\partial t}\hat{\mathbf{B}}(\mathbf{r},t), \quad \nabla \times \hat{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial}{\partial t}\hat{\mathbf{E}}(\mathbf{r},t) \quad (6.a\text{-}d)$$
$$\nabla\cdot\hat{\mathbf{E}}(\mathbf{r},t) = 0, \quad \nabla\cdot\hat{\mathbf{B}}(\mathbf{r},t) = 0$$

Since the time dependence of all terms in (1) is $e^{-i\omega t}$, one can substitute (1) into (4.a) and integrate with respect to time to obtain:

$$\hat{\mathbf{B}}(\mathbf{r},t) = \frac{1}{(2\pi)^2} \sum_{s=1}^{2} \int \sqrt{\frac{h}{2ck\varepsilon_0}} \left[\hat{a}(\mathbf{k},s)\left(\nabla \times \boldsymbol{\varepsilon}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right) + h.c. \right] d^3k \quad (7)$$

Because the only spatial dependence in the free field is $e^{i\mathbf{k}\cdot\mathbf{r}}$, the $\nabla \times \boldsymbol{\varepsilon}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ term can be replaced by $i\mathbf{k} \times \boldsymbol{\varepsilon}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ Thus, in the free field, the magnetic field operator is:

$$\hat{\mathbf{B}}(\mathbf{r},t) = \frac{i}{(2\pi)^2} \sum_{s=1}^{2} \int \sqrt{\frac{h}{2ck\varepsilon_0}} \left[\hat{a}(\mathbf{k},s)\left(\mathbf{k} \times \boldsymbol{\varepsilon}(\mathbf{k},s)\right) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - h.c. \right] d^3k \quad (8)$$

The Heisenberg picture, in which the initial state is fixed and it is the operators that evolve in time, is chosen for the present analysis. It is assumed that the field is initially (at time) t=0 in the free-field state; thus (3) and (8) give the initial values of the electric and magnetic field operators. The operators then evolve according to the Heisenberg equations of motion. However, these are equivalent to the Maxwell operator equations, (6), (see [18], Art. 10.4.5).

As (3) and (8) indicate, to determine the electric field operator beyond t=0, we need only consider the time evolution of the operator $\hat{a}(\mathbf{k},s)\boldsymbol{\varepsilon}(\mathbf{k},s) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$. Hence beyond t=0 , (3) and (8) become:

$$\hat{\mathbf{E}}(\mathbf{r},t) = \frac{i}{(2\pi)^2} \sum_{s=1}^{2} \int \sqrt{\frac{h\hat{\omega}}{2\varepsilon_0}} \left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\hat{a}(\mathbf{k},s) - h.c. \right] d^3k$$
$$\hat{\mathbf{B}}(\mathbf{r},t) = \frac{1}{(2\pi)^2} \sum_{s=1}^{2} \int \sqrt{\frac{h}{2\hat{\omega}\varepsilon_0}} \left[\left(\nabla \times \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\hat{a}(\mathbf{k},s) + h.c.\right) \right] d^3k \quad (9)$$

$\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)$ is a vector-valued function satisfying the wave equation derived from (6), all boundary conditions for , and the initial condition $\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t=0) = \boldsymbol{\varepsilon}(\mathbf{k},s)e^{i(\mathbf{k}\cdot\mathbf{r})}$. $\hat{\omega}(\mathbf{r},t)$ represents the evolution of the initial wave vector to its value at the space-time point (r,t).

The Lorentz force operator per unit volume on the field, **f,** is the primary object of attention:

$$\hat{\mathbf{f}} = \varepsilon_0\left[\left(\nabla\cdot\hat{\mathbf{E}}\right)\hat{\mathbf{E}}+\left(\hat{\mathbf{E}}\cdot\nabla\right)\hat{\mathbf{E}}\right]+\frac{1}{\mu_0}\left[\left(\nabla\cdot\hat{\mathbf{B}}\right)\hat{\mathbf{B}}+\left(\hat{\mathbf{B}}\cdot\nabla\right)\hat{\mathbf{B}}\right]-\frac{1}{2}\nabla\left(\varepsilon_0\hat{E}^2+\frac{1}{\mu_0}\hat{B}^2\right)-\varepsilon_0\mu_0\frac{\partial\hat{\mathbf{S}}}{\partial t}$$

where $\hat{\mathbf{S}}$ is the symmetrized Poynting vector operator:

$$\hat{\mathbf{S}} = \frac{1}{2\mu_0}\left[\hat{\mathbf{E}}(\mathbf{r},t)\times\hat{\mathbf{B}}(\mathbf{r},t)-\hat{\mathbf{B}}(\mathbf{r},t)\times\hat{\mathbf{E}}(\mathbf{r},t)\right] \quad (11)$$

Define volume $V_{R,\delta}$ enclosed by surface $S_{R,\delta}$ as in Figure 3. Integration of (11) over this volume and use of the divergence theorem, produces:

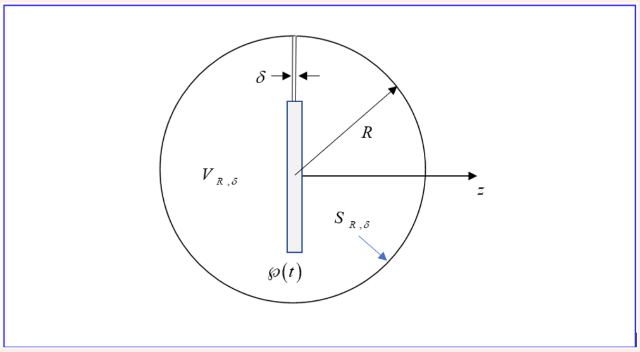
$$\hat{\mathbf{F}}_F(t) = \int_{V_{R,\delta}}\hat{\mathbf{f}}\,dr^3 = \frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\int_{V_{R,\delta}}\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{V_{R,\delta}}\,dS_{R,\delta}-\varepsilon_0\mu_0\frac{d}{dt}\int_{V_{R,\delta}}\hat{\mathbf{S}}\,dr^3 \quad (12)$$

where $\hat{\underline{\boldsymbol{\sigma}}}$ is the Maxwell tensor operator:

$$\left(\hat{\underline{\boldsymbol{\sigma}}}\right)_{ij} = \varepsilon_0\left[\left(\hat{E}_i\hat{E}_j-\tfrac{1}{2}\delta_{ij}\hat{E}^2\right)+c^2\left(\hat{B}_i\hat{B}_j-\tfrac{1}{2}\delta_{ij}\hat{B}^2\right)\right] \quad (13)$$

and where $\int_0^{ct}d\tau\int_{V_{R,\delta}}\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{V_{R,\delta}}\,dS_{R,\delta}$ is the total momentum produced by the stress operator up to time *t*.

Figure 3: Geometry of the volume of integration.



Letting $R\to\infty$ and $\delta\to 0$ one can appreciate that the volume integral extends over the entire space, excluding only the energized lamina (parallel to the x-y plane), and the surface integral is to be taken over the reflective lamina surface, which is treated as of infinitesimal thickness. Therefore:

$$\hat{\mathbf{F}}_F(t) = \frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\int_{\wp(t)}\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{\wp(t)}\,dS_\wp-\varepsilon_0\mu_0\frac{d}{dt}\int_{-\infty}^{\infty}\hat{\mathbf{S}}\,d^3r \quad (14)$$

where $\mathbf{n}_{\wp(t)}$ is the unit normal vector to each surface of the reflective lamina, pointing inward.

A considerable simplification is achieved by recognizing the consequences of the planar surface maintained parallel to the x-y plane, motion solely in the z direction and the homogeneity and isotropy of the reflection coefficient. Further, the free field comprises field modes of all possible wave vectors and polarization states. These characteristics ensure that the force acting on the field must have only a z-component. Thus:

$$\hat{F}_{Fz} = \hat{\mathbf{z}}\cdot\hat{\mathbf{F}}_F = \frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\int_{\wp(t)}\hat{\mathbf{z}}\cdot\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{\wp(t)}\,dS_\wp-\varepsilon_0\mu_0\frac{d}{dt}\int\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}\,d^3r \quad (15)$$

Finally, the average force on the reflective surface, denoted by $\langle F_z\rangle$, is:

$$\langle F_z\rangle = -\frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\int_{\wp(t)}\langle vac|\hat{\mathbf{z}}\cdot\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{\wp(t)}|vac\rangle\,dS_\wp+\varepsilon_0\mu_0\frac{d}{dt}\int\langle vac|\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}|vac\rangle\,d^3r \quad (16)$$

Evaluation of the Average Force in Terms of $\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)$

First, consider the second term in (16). Substituting (11), and (9):

$$\varepsilon_0\mu_0\frac{d}{dt}\int\langle vac|\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}|vac\rangle d^3r =$$

$$-\mathbf{z}\cdot\frac{d}{dt}\frac{ih}{4(2\pi)^3}\int\sum_{s=1}^{2}\sum_{s'=1}^{2}\int d^3k\int d^3k'\sqrt{\frac{\hat{\omega}}{\hat{\omega}'}}\langle vac|\begin{bmatrix}\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}(\mathbf{k},s)\hat{a}(\mathbf{k}',s')\\ +\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s')\\ -\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},t)\times\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},t)\hat{a}^\dagger(\mathbf{k},s)\hat{a}(\mathbf{k}',s')\\ -\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},t)\times\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k}',s'}(\mathbf{r},t)\hat{a}^\dagger(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s')\\ -\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}(\mathbf{k}',s')\hat{a}(\mathbf{k},s)\\ +\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}(\mathbf{k}',s')\hat{a}^\dagger(\mathbf{k},s)\\ -\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}^\dagger(\mathbf{k}',s')\hat{a}(\mathbf{k},s)\\ +\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k}',s'}(\mathbf{r},t)\right)\hat{a}^\dagger(\mathbf{k}',s')\hat{a}^\dagger(\mathbf{k},s)\end{bmatrix}|vac\rangle d^3r \quad (17)$$

Since $\hat{a}(k)|vac\rangle = \langle vac|\hat{a}^\dagger(k) = 0$, the only non-vanishing portion is:

$$\varepsilon_0\mu_0\frac{d}{dt}\int\langle vac|\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}|vac\rangle d^3r$$

$$= \hat{\mathbf{z}}\cdot\frac{d}{dt}\frac{ih}{4(2\pi)^3}\int\sum_{s=1}^{2}\sum_{s'=1}^{2}\int d^3k\int d^3k'\sqrt{\frac{\hat{\omega}}{\hat{\omega}'}}\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k}',s'}(\mathbf{r},t)\right)-\boldsymbol{\Phi}^*_{\mathbf{k}',s'}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]\langle vac|\hat{a}(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s')|vac\rangle d^3r \quad (18)$$

Using the commutation relations, one obtains:

$$\varepsilon_0\mu_0\frac{d}{dt}\int\langle vac|\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}|vac\rangle d^3r = -\hat{\mathbf{z}}\frac{d}{dt}\frac{ih}{4(2\pi)^3}\sum_{s=1}^{2}\int d^3r\int d^3k\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},t)\right)-\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right] \quad (19)$$

Or, more concisely:

$$\varepsilon_0\mu_0\frac{d}{dt}\int\langle vac|\hat{\mathbf{z}}\cdot\hat{\mathbf{S}}|vac\rangle d^3r = \hat{\mathbf{z}}\cdot\frac{d}{dt}\frac{h}{2(2\pi)^3}\sum_{s=1}^{2}\int d^3r\int d^3k\,\mathrm{Im}\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},t)\right)\right] \quad (20)$$

Next, the stress tensor term is considered. In the first term in (16), $\mathbf{n}_{\wp(t)}$ is $-\hat{\mathbf{z}}$ for $z = q(t)+\zeta$, and $+\hat{\mathbf{z}}$ for $z = q(t)-\zeta$ Hence this term can be written:

$$-\frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\int_{\wp(t)}\langle vac|\hat{\mathbf{z}}\cdot\hat{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{n}_{\wp(t)}|vac\rangle dS_\wp = \frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\left[A\langle vac|\hat{\underline{\boldsymbol{\sigma}}}_{zz}|vac\rangle_{z=q+}-\varepsilon_0 A\langle vac|\hat{\underline{\boldsymbol{\sigma}}}_{zz}|vac\rangle_{z=q+}\right] \quad (21)$$

$$= \tfrac{1}{2}A\frac{\varepsilon_0}{c}\frac{d}{dt}\int_0^{ct}d\tau\sum_{S=1,-1}S\langle vac|-\left(\hat{E}_x^2+\hat{E}_y^2-\hat{E}_z^2\right)-c^2\left(\hat{B}_x^2+\hat{B}_y^2-\hat{B}_z^2\right)|vac\rangle_{z=q+S\zeta}$$

where in the last line, (13) has been used to evaluate $\left(\hat{\underline{\boldsymbol{\sigma}}}\right)_{zz}$ and the thickness, ζ, has been assumed several wavelengths beyond the skin layer. Hence are not zero $\left(\hat{E}_x\right)_{z=q+s\zeta}$, and $\left(\hat{E}_y\right)_{z=q+s\zeta}$. Each portion of the integrand is evaluated separately, as follows:

$$\langle vac|\hat{E}_z^2|vac\rangle = \frac{hc}{2(2\pi)^4\varepsilon_0}\sum_{s=1}^{2}\int\hat{k}\left|\hat{\mathbf{z}}\cdot\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2d^3k$$

$$\langle vac|\hat{E}_x^2+\hat{E}_y^2|vac\rangle = \frac{hc}{2(2\pi)^4\varepsilon_0}\sum_{s=1}^{2}\int\hat{k}\left|\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2d^3k-\langle vac|\hat{E}_z^2|vac\rangle \quad \text{(22.a-d)}$$

$$c^2\langle vac|\hat{B}_z^2|vac\rangle = -\frac{hc}{2(2\pi)^4\varepsilon_0}\sum_{s=1}^{2}\int\frac{1}{\hat{k}}\left|\hat{\mathbf{z}}\cdot\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2d^3k$$

$$c^2\langle vac|\hat{B}_x^2+\hat{B}_y^2|vac\rangle = -\frac{hc}{2(2\pi)^4\varepsilon_0}\sum_{s=1}^{2}\int\frac{1}{\hat{k}}\left|\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2d^3k-c^2\langle vac|\hat{B}_z^2|vac\rangle$$

where $\hat{k} = \hat{\omega}(\mathbf{r},t)/c$ represents the evolution of the initial wave vector to its value at the space-time point ***(r,t)***. Collecting results, the following expression is constructed:

$$\langle F_z\rangle = \hat{\mathbf{z}}\cdot\frac{d}{dt}\frac{h}{2(2\pi)^3}\sum_{s=1}^{2}\int d^3r\int d^3k\,R(k)\,\mathrm{Im}\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\boldsymbol{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]$$

$$-\tfrac{1}{2}A\frac{h}{2(2\pi)^4}\frac{d}{dt}\int_0^{ct}d\tau\sum_{S=1,-1}S\sum_{s=1}^{2}\begin{bmatrix}\int\hat{k}\left\{\left|\hat{\mathbf{x}}\cdot\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2+\left|\hat{\mathbf{y}}\cdot\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2-\left|\hat{\mathbf{z}}\cdot\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2\right\}_{z=q+S\zeta}R(k)\,d^3k\\ +\int\frac{1}{\hat{k}}\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2-\left|\hat{\mathbf{z}}\cdot\left(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{z=q+S\zeta}R(k)\,d^3k\end{bmatrix} \quad (23)$$

Decoupling Directivity and Propagation – The 1-D Case

Recall that $\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)$ represents the evolution of the electric field operator from the initial plane wave configuration in the vacuum state having wave vector **K.** For each half space $z > q + \zeta$ and $z < q - \zeta$ there is an incident wave with wave vector and a reflected wave, also planar. Since the objective is to determine such forces as are produced by a reflective surface motion that is much larger than the wavelengths involved, this paper assumes that (1) the total amplitude of motion is much larger than a wavelength, (2) during the time required for the passage of one wavelength, the relative change in the surface velocity is very small. That assumption (2) is compatible with (1) is discussed below in connection with a particular class of motions. With the above assumptions, consider the angular distribution of the integrands in the above expression for $\langle F_z \rangle$ with a view toward simplifying the calculation.

To accomplish this, define spherical coordinates for the **K** space. In each half of position space, the polar axis is taken parallel to the z-axis, pointing toward the reflective surface for the incident wave and the opposite for the reflected wave. Denote the azimuth angle by ϕ and the co-latitude angle by θ First consider the **K** space integral in the first term of $\langle F_z \rangle$:

$$\begin{aligned}&\hat{\mathbf{z}}\cdot\int d^3k\, R(k)\,\mathrm{Im}\left[\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]\\&=\int_0^{2\pi}d\phi\int_0^{\pi/2}\sin\theta\, d\theta\int_0^{\bar{k}}dkR(k)k^2\,\mathrm{Im}\left[\hat{\mathbf{z}}\cdot\left(\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)\right)\right]\end{aligned} \quad (24)$$

Because of assumption (1), cross-products of the incident and reflected waves contribute very little to the spatial integral, hence the quantity $\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)$ is parallel to the wave vector of either incident or reflected wave, independently of polarization. At least in the non-relativistic approximation, the wave vectors of both incident and reflected waves are inclined by angle θ to the z-axis. Hence:

$$\begin{aligned}&\hat{\mathbf{z}}\cdot\int d^3kR(k)\,\mathrm{Im}\left[\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]\\&\cong 2\pi\int_0^{\pi/2}\sin\theta\cos\theta d\theta\int_0^{\bar{k}}dk\, R(k)k^2\,\mathrm{Im}\left[\hat{\mathbf{z}}\cdot\left(\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)\right)_{\mathbf{k}=\pm k\hat{\mathbf{z}}}\right]\\&=\pi\int_0^{\bar{k}}dk\, R(k)k^2\,\mathrm{Im}\left[\hat{\mathbf{z}}\cdot\left(\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)\right)_{\mathbf{k}=\pm k\hat{\mathbf{z}}}\right]\end{aligned} \quad (25)$$

Similarly, in the relativistic case, the integral is distinctly weighted near $\theta = 0$. Regarding the remaining integrals, if one averages over the uniformly distributed polarization vectors, the quantity $\sum_{s=1}^{2}\int k\left|\hat{\mathbf{z}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2_{z=q+S\zeta}R(k)d^3k$ has the angular dependence $\frac{1}{2}\sin^2\theta$

$$\sum_{s=1}^{2}\int k\left|\hat{\mathbf{z}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2_{z=q+S\zeta}R(k)d^3k=\int_0^{2\pi}d\phi\int_0^{\pi/2}\sin^3\theta\, d\theta\int_0^{\bar{k}}dk\, R(k)k^3\max\left|\hat{\mathbf{z}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2_{z=q+S\zeta} \quad (26)$$

thus:

In this case, the integrand is weighted toward the wave vectors with large inclinations to the z-axis. But by symmetry, these contributions to the force are negligible. Similarly, the integrals involving $\left|\hat{\mathbf{z}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2$ and $\left|\hat{\mathbf{z}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2$ can be neglected. Finally, treating the remaining terms in the same fashion as the first term:

$$\begin{aligned}&\int\frac{1}{k}\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{z=q+S\zeta}R(k)d^3k\\&=\int_0^{2\pi}d\phi\int_0^{\pi/2}\sin\theta\, d\theta\int_0^{\bar{k}}dkR(k)k\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{z=q+S\zeta}\\&=2\pi\int_0^{\pi/2}\sin\theta\cos^2\theta\, d\theta\int_0^{\bar{k}}dkR(k)k\max_{\mathbf{k}}\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{z=q+S\zeta}\\&\cong\pi\int_0^{\bar{k}}dk\, R(k)k\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{\mathbf{k}=\pm k\hat{\mathbf{z}}}\end{aligned} \quad (27)$$

In view of the above results, it is reasonable to approximate the electromagnetic field

$$\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)=\Phi_{\mathbf{k}}(\mathbf{r},t)\boldsymbol{\varepsilon}(\mathbf{k},s) \quad (28)$$

involved as one propagating along the z axis. Consequently, one can set:

where $\Phi_{\mathbf{k}}(\mathbf{r},t)$ is a scalar function and $\boldsymbol{\varepsilon}(\mathbf{k},s)$ is in the x-y plane. Then we have the identities:

$$\begin{aligned}&\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{*}_{\mathbf{k},s}(\mathbf{r},t)\right)=\Phi_{\mathbf{k}}(\mathbf{r},t)\nabla\Phi^{*}_{\mathbf{k}}(\mathbf{r},t)\\&\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2=\left|\frac{\partial}{\partial z}\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2\end{aligned} \quad (29)$$

Then, collecting results produces:

$$\begin{aligned}\langle F_z\rangle&\cong A\frac{\hbar c}{2\pi}\frac{d}{dct}\int_0^{\infty}dk\, R(k)k^2\int dz\,\mathrm{Im}\left[\Phi_{\mathbf{k}}(\mathbf{r},t)\nabla\Phi^{*}_{\mathbf{k}}(\mathbf{r},t)\right]\\&-A\frac{\hbar c}{16(\pi)^2}\frac{d}{dct}\int_0^{ct}d\tau\sum_{S=1,-1}S\int_0^{\infty}dk\, R(k)k^2\left\{\hat{k}\left|\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2+\frac{1}{\hat{k}}\left|\frac{\partial}{\partial z}\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2\right\}_{z=q+S\zeta}\end{aligned} \quad (30)$$

Evolution of the Field Modes, $\Phi_{\mathbf{k}}(z,t)$

From the foregoing simplifications, it is clear that field amplitudes must be computed in two distinct half-spaces in the setting of a one-dimensional propagation problem. To adopt more precise notation, let:

$$\Phi_{\alpha k}(z,\tau)=\begin{cases}\Phi_{\mathbf{k}=+k\hat{\mathbf{z}}}(z,\tau),\ \alpha=+1, z<q(\tau)-\zeta\\ \Phi_{\mathbf{k}=-k\hat{\mathbf{z}}}(z,\tau),\ \alpha=-1, z>q(\tau)+\zeta\end{cases} \quad (31)$$

where in this and what follows, $\tau = ct$. The $\Phi_{\mathbf{k}}(\mathbf{r},\tau)$ must satisfy:

$$\begin{aligned}&\frac{\partial^2}{\partial z^2}\Phi_{\alpha k}(z,\tau)=\frac{\partial^2}{\partial\tau^2}\Phi_{\alpha k}(z,\tau)\\&\Phi_{\alpha k}(z,\tau=0)=e^{i\alpha kz}\end{aligned} \quad (32.a,b)$$

The boundary conditions are:

$$\Phi_{\alpha k}\left(z=q(\tau),\tau\right)=0,\ \alpha=\pm1 \quad (33)$$

Suppose that the surface $\wp=(x,y)$ is created (turned on) at location $z=0$ at time $\tau=0$ then travels in the positive z direction with displacement $q(\tau)$ until it reaches $z=\bar{Z}$ time T at which point it is annihilated (turned off). For further reference, let $V(\tau)$ denote the continuous portion of $dq/d\tau$ Obviously, for $|z|>\tau$, the field is undisturbed and thus, by (33), must have the form:

$$\Phi_{\alpha k}(z,\tau)=\exp\left(ik(\alpha z-\tau)\right),\ \alpha=-1,+1 \quad (34)$$

which contributes nothing to the average force.

Consider first the solution to the wave equation for $\Phi_{+1,k}(z,\tau)$ This quantity is the evolution of the field when it is initially in the single mode $\Phi_{+1,k}(z,\tau=0)=\exp(ikz)$ It therefore has both rightward and leftward traveling waves. Since there is a discontinuity at $z=q(t)$, $\Phi_{+1,k}(z,\tau)$, and $\Phi_{-1,k}(z,\tau)$ have the form:

$$\begin{aligned}&\Phi_{+1,k}(z,\tau)=\begin{cases}\exp\left(ik(z-\tau)\right)-\exp\left(ikS_+(-z-\tau)\right), z\le q(\tau)\\ 0,\ \tau\ge z>q(\tau)\end{cases}\\&\Phi_{-1,k}(z,\tau)=\begin{cases}\exp\left(ik(-z-\tau)\right)-\exp\left(ikS_-(z-\tau)\right), z\ge q(\tau)\\ 0,-\tau\le z<q(\tau)\end{cases}\end{aligned} \quad (35.a,b)$$

where the quantities S_+, and S_- are determined by:

$$S_+\left(-q(\tau)-\tau\right)=q(\tau)-\tau,\quad S_-\left(q(\tau)-\tau\right)=-q(\tau)-\tau \quad (36)$$

Since $|q(t)|<ct$, $S_\pm$ can be evaluated by the following sequence (convergent for all):

$$\begin{aligned}&S_\pm(\xi)=\xi\pm2\lim_{k\to\infty}q\left(\xi_{k,\pm}\right)\\&\xi_{k,\pm}=-\xi\mp q\left(\xi_{k-1}\right),\ k\ge1\\&\xi_0=-\xi\end{aligned} \quad (37.a\text{-}c)$$

One should also note the identities:

$$\begin{aligned}&S_+(0)=0\qquad\lim_{\xi\to-2\tau}\left\{S_+(\xi)\right\}=0\\&S_-(0)=0\qquad\lim_{\xi\to0-}\left\{S_-(\xi)\right\}=-2\tau\end{aligned} \quad (38)$$

Determination of the Average Force

Evaluation of the poynting vector term

In the notation of (31), introduced above, the first term given for the average force takes the form:

$$A\frac{\hbar}{2\pi}\frac{d}{dt}\int_0^\infty dk\,R(k)k^2\int dz\;\mathrm{Im}\left[\Phi_{\mathbf{k}}(\mathbf{r},t)\nabla\Phi_{\mathbf{k}}^*(\mathbf{r},t)\right]$$
$$=A\frac{\hbar}{2\pi}\frac{d}{dt}\int_0^\infty dk\,R(k)k^2\left\{\sum_{\alpha=-1}^{+1}\int_{-\infty}^{\infty}\mathrm{Im}\left[\Phi_{\alpha k}(z,\tau)\frac{\partial}{\partial z}\Phi_{\alpha k}^*(z,\tau)\right]dz\right\} \quad (39)$$

It can be shown (see Appendix B) that the spatial integral in braces is:

$$\sum_{\alpha=-1}^{+1}\int_{-\infty}^{\infty}\mathrm{Im}\left[\Phi_{\alpha k}(z,\tau)\frac{\partial}{\partial z}\Phi_{\alpha k}^*(z,\tau)\right]dz=-k\int_0^{-q(\tau)-\tau}d\chi_+\left[1-\cos\left(k\left(\chi_+-2\tau-S_+(\chi_+)\right)\right)\right]\left[\frac{\partial S_+(\chi_+)}{\partial\chi_+}-1\right]$$
$$+k\int_{q(\tau)-\tau}^{0}d\chi_-\left[1-\cos\left(k\left(\chi_-+2\tau-S_-(\chi_-)\right)\right)\right]\left[\frac{\partial S_-(\chi_-)}{\partial\chi_-}-1\right] \quad (40)$$

By assumption (1), after a brief initial transient, $\tau\pm q(\tau)\ll 2\pi/k$ so that the cosine terms (with twice the spatial frequency of the incident waves) contribute very little to the integral. This serves to illustrate the eikonal approximation motivated by assumption (1).

In any case, letting $\Gamma_\pm=S_\pm-\chi_\pm$ and using the relations $S_\pm(\mp q(\tau)-\tau)=\pm q(\tau)-\tau$ one can show that the above integral is identically zero. Thus, the Poynting term vanishes.

Evaluation of the radiation term

The second term in the average force has the form:

$$A\frac{\hbar c}{16(\pi)^2}\frac{d}{d\tau}\int_0^{ct}d\tau\sum_{S=1,-1}S\int_0^\infty dk\,R(k)k^2\left[\hat{k}\left|\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2-\frac{1}{\hat{k}}\left|\frac{\partial}{\partial z}\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2\right]_{z=q+S\zeta}$$
$$\cong A\frac{\hbar c}{16(\pi)^2}\int_0^\infty dk\,R(k)k^3\,\frac{d}{d\tau}\left\{\int_{-\tau}^{q}d\tilde{\tau}\left[1+S_+'(-q-\tilde{\tau})\right]-\int_q^{\tau}d\tilde{\tau}\left[1+S_-'(q-\tilde{\tau})\right]\right\} \quad (41)$$

Differentiating the relations $S_+(-q(\tau)-\tau)=q(\tau)-\tau,\;\; S_-(q(\tau)-\tau)=-q(\tau)-\tau$ one mat determine that:

$$S_+'(-q(\tau)-\tau)=\frac{1-V(\tau)}{1+V(\tau)},\quad S_-'(q(\tau)-\tau)=\frac{1+V(\tau)}{1-V(\tau)} \quad (42)$$

where $V(\tau)=dq(\tau)/d\tau$.

Inserting these into (41):

$$-A\frac{\hbar c}{16(\pi)^2}\frac{d}{d\tau}\int_0^{ct}d\tau\sum_{S=1,-1}S\int_0^\infty dk\,R(k)k\left|\frac{\partial}{\partial z}\Phi_{\mathbf{k}}(\mathbf{r},t)\right|^2_{z=q+S\zeta}$$
$$=A\frac{\hbar c}{16(\pi)^2}\int_0^\infty dk\,R(k)k^3\,\frac{d}{d\tau}\left\{\int_{-\tau}^{q}d\tilde{\tau}\left[1+S_+'(-q-\tilde{\tau})\right]-\int_q^{\tau}d\tilde{\tau}\left[1+S_-'(q-\tilde{\tau})\right]\right\}$$
$$=A\frac{\hbar c}{8(\pi)^2}\int_0^\infty dk\,R(k)k^3\,\frac{d}{d\tau}\left\{\int_{-\tau}^{q}d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})}-\int_q^{\tau}d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \quad (43)$$

As this is the only remaining non-zero term, it follows that the average force per unit area is:

$$\langle F_z\rangle/A=\frac{\hbar c}{8\pi^2}\int_0^\infty dk\,R(k)k^3\,\frac{d}{d\tau}\left(q(\tau)\Lambda(\tau)\right)$$
$$\Lambda(\tau)=\frac{1}{q(\tau)}\left\{\int_{-\tau}^{q(\tau)}d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})}-\int_{q(\tau)}^{\tau}d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \quad (44)$$

Obviously, when the velocity is constant, the force vanishes.

Examples of Various Motion Cases

Note that if the velocity is constant, then as expected, the force is identically zero. Thus, some acceleration of the reflective surface is needed. To study a family of simple examples, consider surface motions that are powers of time:

$$q(\tau)=\bar{Z}(\tau/T)^N,\;\tau\in[0,T] \quad (45)$$

The reflective surface is turned "on" at time zero, accelerates according to an integral power, N, of time, until time T at which it reaches its maximum displacement, $\bar{Z}$ and is turned off. Note that both $\bar{Z}$ and T have the dimension of length. The velocity may be written:

$$V(\tau)=\bar{V}(\tau/T)^{N-1},\;\bar{V}=N\bar{Z}/T \quad (46)$$

It is seen that $\bar{V}$ is the maximum attainable velocity during the maneuver and is expressed as a fraction of the speed of light.

The first object for computation is the quantity $\Lambda(\tau=T)$ which epitomizes the asymmetry in the field due to the reflective surface motion. Substituting the above expressions:

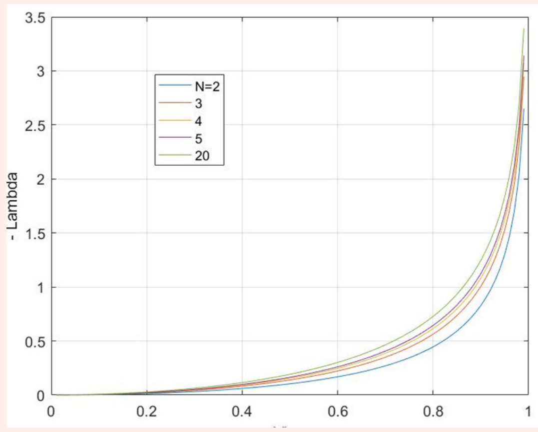
$$\Lambda(\tau=T)=\frac{1}{\bar{Z}}\left\{\int_{-T}^{\bar{Z}}d\tilde{\tau}\frac{1}{1+\bar{V}\left(\frac{T+\tilde{\tau}}{T+\bar{Z}}\right)^{N-1}}-\int_{\bar{Z}}^{T}d\tilde{\tau}\frac{1}{1-\bar{V}\left(\frac{T-\tilde{\tau}}{T-\bar{Z}}\right)^{N-1}}\right\} \quad (47)$$

Clearly for $N=1$ the above quantity vanishes (Figure 4). To evaluate it explicitly for $N>1$ the two integrands are expanded as geometric series, then the integration performed term-by-term, with the following result:

$$\Lambda(\tau=T)=\sum_{m=0}^{\infty}\frac{\bar{V}^m}{m(N-1)+1}\left[(-1)^m\left(\frac{N}{\bar{V}}+1\right)-\left(\frac{N}{\bar{V}}-1\right)\right] \quad (48)$$

Thus $\Lambda(\tau=T)$ is solely a function of $\bar{V}$ and independent of $\bar{Z}$ Fig. 4 shows the behavior of $\Lambda(\tau=T)$ as $\bar{V}$ increases for various values of N. For $\bar{V}$ close to unity, Λ begins to diverge. For large $N,\Lambda(\tau=T)$ approaches an asymptotic limit (approximated here by N=20).

Figure 4: The Lambda integral versus maximum speed.



At this point, one might pause to consider the validity of assumption (2) given the condition $\tau\pm q(\tau)\ll 2\pi/k$ implied by assumption (1). The time required for a wave with initial wave number k, to travel one wavelength is $2\pi/kc$ Under the current motion model, the change in the normalized velocity in that time interval, denoted by ΔV is given by:

$$\frac{\Delta V}{\bar{V}}=(N-1)\frac{\lambda}{T}\left(\frac{\tau}{T}\right)^{N-2} \quad (49)$$

In other words, the change of velocity over one cycle relative to the maximum speed is of the order of the ratio of the wavelength to the distance light travels over the entire duration of the motion. Since this analysis considers wavelengths of the order of microns, and motion sequences covering centimeters, both assumptions (1) and (2) are well justified.

Consideration of the Three-Dimensional Problem

Now consider the more general formulation, Equation (23):

$$\langle F_z \rangle = \hat{\mathbf{z}}\cdot\frac{d}{dt}\frac{h}{2(2\pi)^3}\sum_{s=1}^{2}\int d^3r\int d^3k\, R(k)\,\mathrm{Im}\left[\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]$$

$$-\tfrac{1}{2}A\frac{h}{2(2\pi)^4}\frac{d}{dt}\int_0^{ct}d\tau\sum_{S=1,-1}S\sum_{s=1}^{2}\begin{bmatrix}\int\hat{k}\left\{\left|\hat{\mathbf{x}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2+\left|\hat{\mathbf{y}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2-\left|\hat{\mathbf{z}}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2\right\}_{z=q+S\zeta}R(k)d^3k\\+\int\frac{1}{\hat{k}}\left\{\left|\hat{\mathbf{x}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2+\left|\hat{\mathbf{y}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2-\left|\hat{\mathbf{z}}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2\right\}_{z=q+S\zeta}R(k)d^3k\end{bmatrix}$$

(50)

One consequence of assumption (2) is that the above quantities may be approximated by their running time averages over periods that include many oscillations, yet are still so short that there is little variation in the reflector surface velocity. Henceforth, it is understood that such averaging is to be applied. In each half space, and for a given incident wave vector, each of the two terms above has three contributions: One involving the incident wave alone, a second involving the reflected wave alone and the third composed of products of the incident and reflected waves. As in the one-dimensional formulation, and assuming the range of motion is much larger than the wavelengths involved, the third has a higher spatial frequency and averages out to a negligible contribution. Since only pairs of waves passing in the same direction survive the time averaging process, and because all waves are planar (but not monochromatic), the field appearing in the force expression is described by the eikonal approximation (for propagation in a homogeneous medium):

$$\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)=\exp\left(i\left(\mathbf{k}(\tau)\cdot\mathbf{r}-k\tau\right)\right)\boldsymbol{\varepsilon}\left(\mathbf{k}(\tau),s\right) \quad (51)$$

Then one obtains:

$$\begin{aligned}&\mathrm{Im}\left[\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\left(\nabla\times\mathbf{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},t)\right)\right]=k\kappa(\mathbf{r},\theta,\tau)\\&\hat{k}\left\{\left|\mathbf{u}\cdot\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2\right\}=\frac{1}{\hat{k}}\left|\mathbf{u}\cdot\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right)\right|^2=k\left|\left(\mathbf{u}-(\hat{\mathbf{x}}\cdot\mathbf{u})\mathbf{u}\right)\cdot\kappa(\mathbf{r},\theta,\tau)\theta\right|\\&\kappa(\mathbf{r},\theta,\tau)=\mathbf{k}(\mathbf{r},\theta,\tau)/k\end{aligned} \quad (52)$$

where $\mathbf{U}$ is any constant unit vector and recall that $\hat{\mathbf{x}}$ is orthogonal to the z axis and in the plane of z and the direction of propagation of the vacuum mode under consideration. Substituting the above results into the force expression (50), noting that the integrands are independent of the polarization vector, and replacing d^3k with $2\pi\sin\theta d\theta k^2dk$ the force becomes:

$$\langle F_z \rangle = A\frac{hc}{(2\pi)^2}\int d^3k\,R(k)k^3\int_0^{\pi/2}d\theta\sin\theta\frac{d}{d\tau}\int dz\,\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tau)$$

$$-A\frac{hc}{(2\pi)^3}\int d^3k\,R(k)k^3\int_0^{\pi/2}d\theta\sin\theta\frac{d}{d\tau}\int_0^{\tau}d\tilde{\tau}\sum_{S=1,-1}S\left[\int\left\{\left|\kappa(\mathbf{r},\theta,\tilde{\tau})\right|-2\left|\hat{\mathbf{x}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q+S\zeta}\right]$$

(53)

In consequence of the assumptions, one can show that the first term vanishes. First, for each inclination angle of the incident waves, no tangential component of force can be produced by specular reflection without violating the idea of relativity. If this were possible, one could measure the force in its rest frame and deduce the reflective surface velocity relative to a nonexistent "ether". Thus, on both sides of the surface, the net momentum change is along the direction of motion. The direction is positive for the left side and negative for the right.

Next consider the change in the spatial integral of the momentum density over a period of many oscillations but during which the velocity change is negligible. Then, in the rest frame, the approximations for uniform motion apply. In this frame, the space-time factor of the vacuum state remains the same, and one can define the inclination angle of the incident waves as having the same amplitude, but opposite directions. Since the rest frame presents negligible asymmetry, the reflected waves must be very nearly symmetric with respect to the surface and have equal and opposite components along the z axis. Therefore, within the approximate formulation, the rate of change of the spatial integral of the Poynting vector is negligible (and, rigorously, identically zero). Likewise, the average energy of the field can be shown to be unchanging.

If β is the angle of reflection corresponding to the angle of incidence, θ, then the absence of a tangential component of force implies that $\kappa(\mathbf{r},\theta,\tau)\sin\beta=k\sin\theta$. This, in turn, permits the conclusion that $\left\{\left|\kappa(\mathbf{r},\theta,\tilde{\tau})\right|-2\left|\hat{\mathbf{x}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}=\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|$ As a result of the foregoing simplifications, the expression for the force becomes:

$$\langle F_z \rangle = -A\frac{hc}{(2\pi)^3}\int d^3k\,R(k)k^3\int_0^{\pi/2}d\theta\sin\theta\frac{d}{d\tau}\int_0^{\tau}d\tilde{\tau}\sum_{S=1,-1}S\left[\int\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q+S\zeta}\right] \quad (54)$$

Let the incident wave vectors both form angle θ with the z axis, and let β_1 and β_2 be the inclination angles of the reflected wave vectors in the regions $z<q$ and $z\geq q$ respectively. Then, considering only the τ integrals for the moment:

$$\begin{aligned}&\int_0^{\tau}d\tilde{\tau}\left\{\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q-\zeta}-\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q+\zeta}\right\}\\&\quad=\int_0^{\tau}d\tilde{\tau}\left(\cos\theta+\left|\kappa_1(\mathbf{r},\theta,\tilde{\tau})\right|\cos\beta_1\right)-\int_0^{\tau}d\tilde{\tau}\left(\cos\theta+\left|\kappa_2(\mathbf{r},\theta,\tilde{\tau})\right|\cos\beta_2\right)\end{aligned} \quad (55)$$

where $\kappa_1(\mathbf{r},\theta,\tilde{\tau})$ and $\kappa_2(\mathbf{r},\theta,\tilde{\tau})$ are the wave vectors of the reflected waves. Next, as above, it is assumed that the variations of the reflected wave amplitudes over several wavelengths are very small. Then the formulae related to the uniform motion of a mirror [20,21] may be used:

$$\begin{aligned}\left|\kappa_1(\mathbf{r},\theta,\tilde{\tau})\right|&=\frac{1-2V\cos\theta+V^2}{1-V^2},\ \cos\beta_1=\frac{-2V+(1+V^2)\cos\theta}{1-2V\cos\theta+V^2}\\\left|\kappa_2(\mathbf{r},\theta,\tilde{\tau})\right|&=\frac{1+2V\cos\theta+V^2}{1-V^2},\ \cos\beta_2=\frac{2V+(1+V^2)\cos\theta}{1+2V\cos\theta+V^2}\end{aligned} \quad (56)$$

Then making substitutions into (55), one concludes:

$$\int_0^{\tau}d\tilde{\tau}\left\{\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q-\zeta}-\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q+\zeta}\right\}=\int_0^{\tau}d\tilde{\tau}\frac{2(\cos\theta-V)}{1-V^2}-\int_0^{\tau}d\tilde{\tau}\frac{2(\cos\theta+V)}{1-V^2} \quad (57)$$

Since the time variable has units of length, one can convert to integration along the z-axis. For the regions $z<\tau\cos\beta_1(\tau=0)+q(\tau)$, only incident waves exist which contribute nothing to the momentum change. Adjusting the limits of integration accordingly, the integrals become:

$$\int_0^{\tau}d\tilde{\tau}\left\{\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q-\zeta}-\left\{\left|\hat{\mathbf{z}}\cdot\kappa(\mathbf{r},\theta,\tilde{\tau})\right|\right\}_{z=q+\zeta}\right\}=\int_{-\chi_1(\tau)}^{q(\tau)}d\tilde{\tau}\frac{\left(\cos\theta-V(\tau+\tilde{\tau})\right)}{1-V^2(\tau+\tilde{\tau})}-\int_{q(\tau)}^{\chi_2(\tau)}d\tilde{\tau}\frac{\left(\cos\theta+V(\tau-\tilde{\tau})\right)}{1-V^2(\tau-\tilde{\tau})} \quad (58)$$

where χ_1 and χ_2 are given by:

$$\frac{d}{d\tau}\chi_1=\max\left\{0,\frac{-2V+(1+V^2)\cos\theta}{1-2V\cos\theta+V^2}\right\},\quad \frac{d}{d\tau}\chi_2=\frac{2V+(1+V^2)\cos\theta}{1+2V\cos\theta+V^2} \quad (59)$$

The result for the force becomes:

$$\langle F_z \rangle = A\frac{\hbar c}{(2\pi)^2}\left(\int_0^{\bar{k}}dk\,R(k)k^3\right)\frac{d}{d\tau}\left[\int_0^{\pi/2}\sin\theta\,d\theta\left\{\int_{-\chi_1(\tau)}^{q(\tau)}d\tilde{\tau}\frac{\left(\cos\theta-V(\tau+\tilde{\tau})\right)}{1-V^2(\tau+\tilde{\tau})}-\int_{q(\tau)}^{\chi_2(\tau)}d\tilde{\tau}\frac{\left(\cos\theta+V(\tau-\tilde{\tau})\right)}{1-V^2(\tau-\tilde{\tau})}\right\}\right] \quad (60)$$

If the velocity is constant, the force is zero. Let $V=\bar{V}=const$ then:

$$\chi_1=\max\left\{0,\frac{-2V+(1+V^2)\cos\theta}{1-2V\cos\theta+V^2}\right\}\tau,\quad \chi_2=\frac{2\bar{V}+(1+\bar{V}^2)\cos\theta}{1+2\bar{V}\cos\theta+\bar{V}^2}\tau \quad (61.a,b)$$

and the integrals in the above force expression become:

$$\begin{aligned}&\int_{-\chi_1(\tau)}^{q(\tau)}d\tilde{\tau}\frac{\left(\cos\theta-V(\tau+\tilde{\tau})\right)}{1-V^2(\tau+\tilde{\tau})}-\int_{q(\tau)}^{\chi_2(\tau)}d\tilde{\tau}\frac{\left(\cos\theta+V(\tau-\tilde{\tau})\right)}{1-V^2(\tau-\tilde{\tau})}\\&=\left(\frac{\cos\theta-\bar{V}}{1-\bar{V}^2}\right)\left(\bar{V}+\frac{(1+\bar{V}^2)\cos\theta-2\bar{V}}{1-2\bar{V}\cos\theta+\bar{V}^2}\right)-\left(\frac{\cos\theta+\bar{V}}{1-\bar{V}^2}\right)\left(-\bar{V}+\frac{(1+\bar{V}^2)\cos\theta+2\bar{V}}{1+2\bar{V}\cos\theta+\bar{V}^2}\right)\\&=\left(\frac{\cos\theta-\bar{V}}{1-\bar{V}^2}\right)\left(\frac{(1-\bar{V}^2)(\cos\theta-\bar{V})}{1-2\bar{V}\cos\theta+\bar{V}^2}\right)-\left(\frac{\cos\theta+\bar{V}}{1-\bar{V}^2}\right)\left(\frac{(1-\bar{V}^2)(\cos\theta+\bar{V})}{1+2\bar{V}\cos\theta+\bar{V}^2}\right)\\&=\left(\frac{(\cos\theta-\bar{V})^2}{1-2\bar{V}\cos\theta+\bar{V}^2}\right)-\left(\frac{(\cos\theta+\bar{V})^2}{1+2\bar{V}\cos\theta+\bar{V}^2}\right)\end{aligned} \quad (62)$$

In the integral of the first term, the sign of the cosine can be reversed. Therefore, the integral of the first term is equal and opposite in sign to that of the second term and the force vanishes.

Approximation for High- and Low-Speed Scans

Since the epitaxial device is intended to both increase the amplitude, and the speed of the reflective surface motion it is well to consider the case where V is a significant fraction of unity [22,23]. Let $V = 1 - \nu,\ \nu << 1$. Then the relations for the reflection angles yield:

$$\cos|\beta_1| = 1 + O(\nu^2),\ \cos\beta_2 = 1 + O(\nu^2) \quad (63.a,b)$$

Secondly, note that $-\frac{(\cos\theta - V)}{1-V^2} \geq -\frac{\cos\theta}{1+V}$, and $\frac{(\cos\theta + V)}{1-V^2} \geq \frac{\cos\theta}{1-V}$ Using these inequalities and the above approximations for the reflection angle cosines, one obtains an upper bound for the spatial integral:

$$\int_{-\chi_1(\tau)}^{q(\tau)} d\tilde{\tau}\frac{(\cos\theta - V(\tau+\tilde{\tau}))}{1-V^2(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\chi_2(\tau)} d\tilde{\tau}\frac{(\cos\theta + V(\tau-\tilde{\tau}))}{1-V^2(\tau+\tilde{\tau})} \leq \cos\theta\left[\int_{-\tau}^{q(\tau)} dz\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} dz\frac{1}{1-V(\tau+\tilde{\tau})}\right] + O(\nu^2) \quad (64)$$

Substituting this into the force expression, and performing the integration over θ yields:

$$\begin{aligned} &\left|\langle F_z\rangle/A\right| \geq \left|\langle \hat{F}_z\rangle/A + O(\nu^2)\right| \\ &\left|\langle \hat{F}_z\rangle/A\right| = \frac{\hbar c}{8\pi^2}\left(\int_0^{\bar{k}} dk R(k) k^3\right)\bar{Z}\left|\frac{d}{d\tau}\hat{\Lambda}(\tau)\right| \\ &\hat{\Lambda}(\tau) = \frac{1}{\bar{Z}}\left\{\int_{-\tau}^{q(\tau)} d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \end{aligned} \quad (65.a\text{-}c)$$

The expression in the bracket obviously vanishes when V is constant. This lower bound is identical to the magnitude of the one-dimensional approximation.

On the other hand, assuming $V<<1$, an upper bound can be discerned:

$$\begin{aligned} &\int_{-\chi_1(\tau)}^{q(\tau)} d\tilde{\tau}\frac{(\cos\theta - V(\tau+\tilde{\tau}))}{1-V^2(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\chi_2(\tau)} d\tilde{\tau}\frac{(\cos\theta + V(\tau-\tilde{\tau}))}{1-V^2(\tau-\tilde{\tau})} \\ &\cong \int_{-\tau\cos\theta}^{q(\tau)} d\tilde{\tau}\frac{(\cos\theta - V(\tau+\tilde{\tau}))}{1-V^2(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau\cos\theta} d\tilde{\tau}\frac{(\cos\theta + V(\tau-\tilde{\tau}))}{1-V^2(\tau+\tilde{\tau})} \\ &\leq \int_{-\tau\cos\theta}^{q(\tau)} d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau\cos\theta} d\tilde{\tau}\frac{1}{1-V(\tau+\tilde{\tau})} \end{aligned} \quad (66)$$

The force expression then yields:

$$\begin{aligned} &\left|\langle F_z\rangle/A\right| = \left|\langle \breve{F}_z\rangle/A\right| + O(V) \\ &\left|\langle \breve{F}_z\rangle/A\right| \leq \frac{2\hbar c}{8\pi^2}\left(\int_0^{\bar{k}} dk R(k) k^3\right)\bar{Z}\left|\frac{d}{d\tau}\breve{\Lambda}(\tau)\right| \\ &\breve{\Lambda}(\tau) = \frac{1}{\bar{Z}}\left\{\int_{-\tau}^{q(\tau)} d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \end{aligned} \quad (67.a\text{-}c)$$

Average Force in the Case of Periodic Scans

One could create a periodic disturbance by repeating the surface displacement waveform. However, if the motion is immediately repeated at the end of a cycle, there will be interaction between the newly created waves and the reverberant wave still crossing the segment $z \in [0, \bar{Z}]$. Such interaction ceases if one waits for time $\bar{Z}/c$ to begin the new cycle. The momentum increment will then be the same for each cycle. The following analysis, adopts the one-dimensional approximation for the Casimir force, (65). Figure 5a shows, schematically, the cyclic surface position waveform so produced. By momentum conservation, the force on the mirror device is the negative derivative of the momentum change, which is proportional to $\Lambda(\tau = T)$. This is illustrated in Figure 5b. Note he negative direction of the force.

Using Equation (65), and setting the total duration of the cycle equal to $(T + \bar{Z})/c$ one concludes:

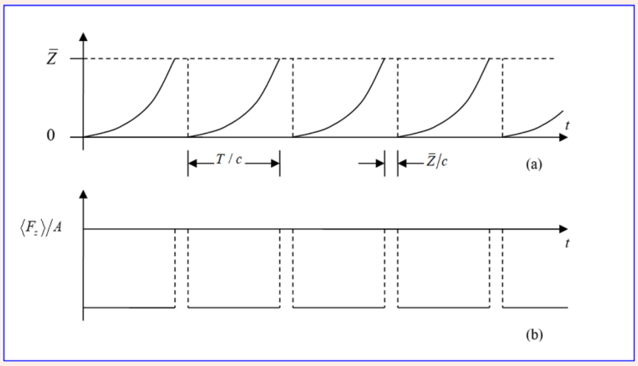
$$\begin{aligned} &\langle\langle F_z\rangle/A\rangle_t = \frac{\hbar c}{8\pi^2}\left(\int_0^{\infty} dk\, R(k) k^3\right)\frac{\bar{Z}}{T+\bar{Z}}\Lambda(T) \\ &\Lambda(T) = \frac{1}{\bar{Z}}\left\{\int_{-T}^{\bar{Z}} d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{\bar{Z}}^{T} d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \end{aligned} \quad (68.a,b)$$

At this point it is well to examine the effect of various parameters on the force produced, and perhaps reformulate their definitions. At the outset, it was assumed that the reflection coefficient was unity (perfect reflection) up to some wave number cutoff beyond which it is zero (perfect transparency). A somewhat more realistic, albeit still crude, model is that:

$$R(k) = \begin{cases} 1, & k \in [k_L, k_U] \\ 0, & otherwise \end{cases} \quad (69)$$

where $k_U > k_L$. With this expression, the wavenumber integral can be seen to be:

Figure 5: (a) Cyclic waveform of the reflective surface position; (b) Force on the epitaxial device.



$$\begin{aligned} &\left(\int_0^{\infty} dk\, R(k) k^3\right) = \bar{\bar{k}}^3 \Delta\bar{k} \\ &\bar{\bar{k}} = \left[\tfrac{1}{2}(k_U^2 + k_L^2)\cdot\tfrac{1}{2}(k_U + k_L)\right]^{1/3} \\ &\Delta\bar{k} = k_U - k_L \end{aligned} \quad (70.a\text{-}c)$$

$\bar{\bar{k}}$ represents the approximate middle (weighted heavily at the upper end) of the useful band wherein reflectivity/transparency can be switched on and off, and $\Delta\bar{k}$ is the width of this band.

Another parameter of interest is $\bar{\beta} = 2\bar{Z}/(T + \bar{Z}) \in [0,1)$ This is the average speed, relative to c, that the activated laminae sweep through the total period of the waveform. Finally, $\Lambda(T)$ is dimensionless, depends only on the normalized velocity profile, and is of order unity unless V is nearly unity. To summarize:

$$\begin{aligned} &\langle\langle F_z\rangle/A\rangle_t = \frac{\hbar c}{(4\pi)^2}\bar{\bar{k}}^3\Delta\bar{k}\,\bar{\beta}\,\Lambda(T) \\ &\Lambda(T) = \frac{1}{\bar{Z}}\left\{\int_{-T}^{\bar{Z}} d\tilde{\tau}\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{\bar{Z}}^{T} d\tilde{\tau}\frac{1}{1-V(\tau-\tilde{\tau})}\right\} \\ &\bar{\bar{k}} = \left[\tfrac{1}{2}(k_U^2 + k_L^2)\cdot\tfrac{1}{2}(k_U + k_L)\right]^{1/3} \\ &\Delta\bar{k} = k_U - k_L \\ &\bar{\beta} = 2\bar{Z}/(T + \bar{Z}) \in [0,1) \end{aligned} \quad (71.a\text{-}e)$$

Example: Periodic Motion with Power Law Waveforms

Here some numerical examples are shown, involving the waveforms proportional to an integer power of time, treated earlier, with $\bar{V}$ denoting the maximum velocity of the reflective surface. To appreciate the magnitude of the force per unit area, assign plausible values to the various parameters. Suppose the plasma frequency is in the range $10^{14}\,Hz$ to $10^{16}\,Hz$. Specifically, let us fix $k_U = 2\times10^7$ $(\lambda \cong 0.3\mu m)$. Hence the average force depends only on the maximum speed and the power law exponent.

Figure 6 shows the variation of the force with maximum speed for integers powers, N=1 to 7. Similar results are shown in Figure 7 for fractional powers between 1.1 and 2. It appears that for any given maximum speed (below c), an approximately quadratic velocity yields the maximum force per unit area.

Figure 6: Force per unit area as a function of the maximum waveform velocity, integer powers.

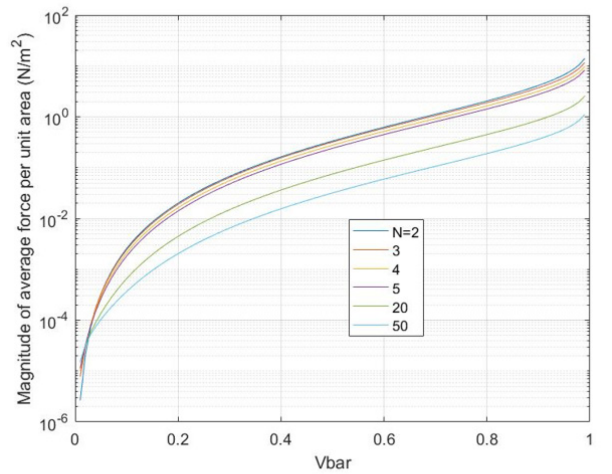
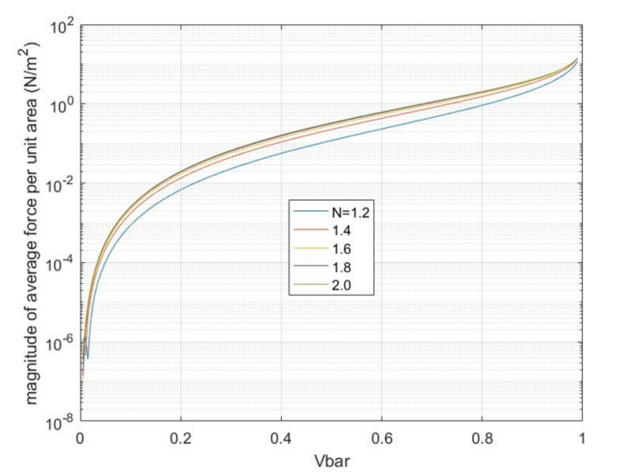


Figure 7: Force per unit area as a function of the maximum waveform velocity, fractional powers.



Concluding Remarks

This paper re-examines the dynamic Casimir effect as a possible mechanism for propulsion. Previous investigations assumed mechanical motion of a mirror to generate thrust. In this case, because of the finite strength of materials and the high frequencies necessary, the amplitudes of motion must be restricted to the nanometer range. Here, an epitaxial stack of transparent semiconductor laminae is proposed, where voltage is rapidly switched to successive laminae, thereby creating continuous motion of a front of charge carrier density. The result is the creation of a reflective surface in rapid, large amplitude motion without the use of mechanical contrivances. Since previous analysis of the propulsive effect was restricted to motions much smaller than the wavelengths of importance, it was necessary to derive correct relativistic expressions appropriate for large amplitude motion. This was accomplished for the general motion case and examined in detail for a variety of possible motions. All calculations assumed an initial zero temperature state. Another restriction is that detailed dielectric function models were not used; rather the reflectivity was based on a simple wave number cut-off model. Moreover, as for previous workers, the treatment is semi-quantum in that the epitaxial stack is modeled as a set of prescribed boundary conditions on the field operators. Despite these restrictions, if reasonable charge carrier volumetric densities are assumed, the propulsive forces may be quite significant. The assumption of finite temperature and surface velocities that are a significant fraction of the light speed may possibly increase the magnitude of the present estimates. With further progress in the use of light rather than rocketry, we may see an expansion of habitable worlds (the Oikoumene) in our solar system.

Back Matter

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Supplementary Materials

Reside in the ICI algorithm as described in full detail within the Review.

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Appendix A

The finite response time of a semiconductor lamina allows us to create a continuously moving "front" at which the cumulative areal density of charge carriers suffices to produce a desired level of reflectance. Thus, although the laminae are discrete, their sequential stimulus at the proper rate yields the effect of a continuously moving mirror.

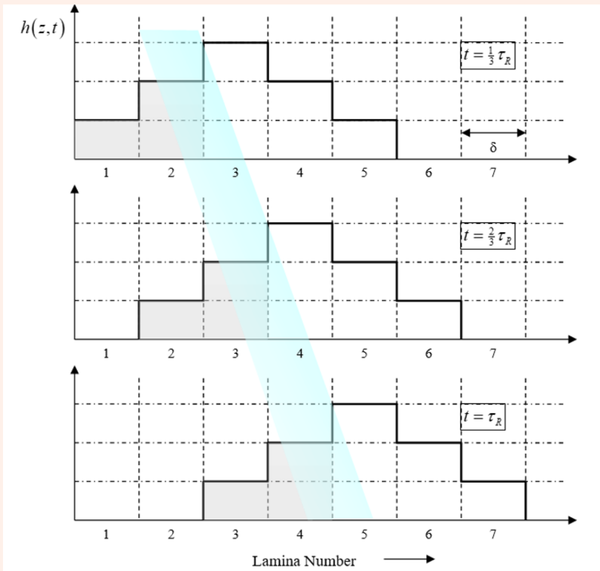
As an illustration, suppose that the conductivity, $h(t)$ in response to a voltage pulse has a simple linear rise and fall, as in:

$$h(t) = \rho_{\max} \begin{cases} t/\tau_R, t \le \tau_R \\ 2 - t/\tau_R, 2\tau_R \ge t \ge \tau_R \\ 0, t > 2\tau_R \end{cases} \quad \text{(A-1)}$$

where τ_R is the finite rise time and $h(t)$ is the impulse response of the lamina conductivity. Suppose each successive lamina is stimulated at a sub-multiple of the rise time after its immediate predecessor, such that each rise in the reflectance is a small fraction of complete reflectivity. Figure A-1 illustrates the resulting motion of the conductivity profile. An incoming plane wave suffers a cumulative reflection in proportion to the total charge carrier population per unit area along its path. In the example of the figure, the total areal population corresponding to some reflection coefficient, $|R(k)|$ is suggested by the gray-shaded areas. In general, the position of the "front" along which the total reflectance reaches some value is seen to move continuously in the direction of, and with the approximate speed of, the conductivity profile (illustrated by the blue-shaded boundary in the Figure). Even if the charge carrier profile has the staircase form as shown in the Figure, the Courant-Friedrichs-Lewy condition can be satisfied so that the effective conductivity profile approximates a continuously increasing distribution. This permits the device to approximate the reflective properties of a mechanical mirror, including the relativistic Doppler effects. Moreover, remaining discretization effects can be mitigated by designing a suitable charge carrier gradient for each lamina. Because of length limitations, detailed proofs of the foregoing results must be deferred to a latter manuscript.

To assess the achievable front speeds, consider the example of Figure A-1 where the time between inputs to successive laminas is a third of the rise time. Then the average speed of the reflective surface is $\sim 3\delta/\tau_R$ where δ is the lamina thickness. Taking a typical rise time of and a lamina thickness of a millimeter, obtains a reflective surface speed of $\sim 3\times10^{-3}\ m/s$, i.e., $\beta \cong 0.1$ This could be significantly improved by advanced high-speed switching technology.

Figure A-1: Temporal progression of conductivity as the laminas are successively pulsed. The blue-shaded boundary indicates the continuous motion of the front having a particular value of reflectance.



Appendix B

$$\sum_{\alpha=-1}^{+1}\int_{-\infty}^{\infty} \mathrm{Im}\left[\Phi_{\alpha k}(z,\tau)\frac{\partial}{\partial z}\Phi^*_{\alpha k}(z,\tau)\right]dz =$$

$$\int_{-\tau}^{q(\tau)} dz\,\mathrm{Im}\left\{\left[\exp(ik(z-\tau)) - \exp(ikS_+(-z-\tau))\right]\left[-ik\exp(-ik(z-\tau)) + ik\frac{\partial S_+(-z-\tau)}{\partial z}\exp(-ikS_+(-z-\tau))\right]\right\}$$

$$+\int_{q(\tau)}^{\tau} dz\,\mathrm{Im}\left\{\left[\exp(ik(-z-\tau)) - \exp(ikS_-(z-\tau))\right]\left[ik\exp(-ik(-z-\tau)) + ik\frac{\partial S_-(z-\tau)}{\partial z}\exp(-ikS_-(z-\tau))\right]\right\}$$

$$= \int_{-\tau}^{q(\tau)} dz\,\mathrm{Im}\left\{-ik + ik\frac{\partial S_+(-z-\tau)}{\partial z}\exp(ik(z-\tau-S_+(-z-\tau))) + ik\exp(-ik(z-\tau-S_+(-z-\tau))) - ik\frac{\partial S_+(-z-\tau)}{\partial z}\right\}$$

$$+\int_{q(\tau)}^{\tau} dz\,\mathrm{Im}\left\{ik + ik\frac{\partial S_-(z-\tau)}{\partial z}\exp(ik(-z-\tau-S_-(z-\tau))) - ik\exp(-ik(-z-\tau-S_-(z-\tau))) - ik\frac{\partial S_-(z-\tau)}{\partial z}\right\}$$

$$= k\int_{-\tau}^{q(\tau)} dz\left\{-1 + \frac{\partial S_+(-z-\tau)}{\partial z}\cos(k(z-\tau-S_+(-z-\tau))) + \cos(k(z-\tau-S_+(-z-\tau))) - \frac{\partial S_+(-z-\tau)}{\partial z}\right\}$$

$$+ k\int_{q(\tau)}^{\tau} dz\left\{1 + \frac{\partial S_-(z-\tau)}{\partial z}\cos(k(z+\tau+S_-(z-\tau))) - \cos(k(z+\tau+S_-(z-\tau))) - \frac{\partial S_-(z-\tau)}{\partial z}\right\}$$

$$= k\int_{-\tau}^{q(\tau)} dz\left[1-\cos(k(z-\tau-S_+(-z-\tau)))\right]\left[\frac{\partial S_+(-z-\tau)}{\partial(-z)} - 1\right]$$

$$- k\int_{q(\tau)}^{\tau} dz\left[1-\cos(k(z+\tau-S_-(z-\tau)))\right]\left[\frac{\partial S_-(z-\tau)}{\partial z} - 1\right]$$

(B-1)

Let:

$$\chi_+ = -z-\tau, \quad \chi_- = z-\tau \quad \text{(B-2)}$$

Then we have:

$$\sum_{\alpha=-1}^{+1}\int_{-\infty}^{\infty} \mathrm{Im}\left[\Phi_{\alpha k}(z,\tau)\frac{\partial}{\partial z}\Phi^*_{\alpha k}(z,\tau)\right]dz$$

$$= k\int_{-\tau}^{q(\tau)} dz\left[1-\cos(k(z-\tau-S_+(-z-\tau)))\right]\left[\frac{\partial S_+(-z-\tau)}{\partial(-z)} - 1\right] \quad \text{(B-3)}$$

$$- k\int_{q(\tau)}^{\tau} dz\left[1-\cos(k(z+\tau-S_-(z-\tau)))\right]\left[\frac{\partial S_-(z-\tau)}{\partial z} - 1\right]$$

$$= -k\int_{0}^{-q(\tau)-\tau} d\chi_+\left[1-\cos(k(\chi_+ - 2\tau - S_+(\chi_+)))\right]\left[\frac{\partial S_+(\chi_+)}{\partial \chi_+} - 1\right]$$

$$+k\int_{q(\tau)-\tau}^{0} d\chi_-\left[1-\cos(k(\chi_- + 2\tau - S_-(\chi_-)))\right]\left[\frac{\partial S_-(\chi_-)}{\partial \chi_-} - 1\right]$$