CORPUS PUBLISHERS

Current Trends in Engineering Science (CTES)

ISSN: 2833-356X

Volume 3 Issue 2, 2023

Article Information

Received date : March 03, 2023

Published date: April 14, 2023

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DOI: 10.54026/CTES/1026

Keywords

Single station location; Passive location; ranging; Doppler change rate; Doppler shift; Path difference

Mini Review

Doppler Ranging Solution Derived Based on the Relationship between Frequency Shift and Path Difference

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Abstract

Based on the mathematical definition of Doppler change rate, a single base Doppler ranging formula based on Doppler shift measurement has been obtained by differential processing. In this paper, the single base Doppler ranging equation is deduced again based on the double base path difference ranging equation and the interchangeable relationship between frequency shift and path difference.

Introduction

For the passive ranging method based on Doppler rate of change, one of the early research results of the author is that the Doppler rate of change is converted into the ratio of Doppler frequency difference to time difference by differential processing directly based on the mathematical definition, and the tangential velocity included in the equation is converted into the radial velocity related to Doppler frequency shift measurement by using the velocity vector relationship, From this, an analytical formula of airborne Doppler passive ranging based on secondary frequency shift measurement is derived [1-3]. This ranging solution based on differential processing does not need to directly detect the change rate of Doppler frequency shift, nor does it need to be used in conjunction with other positioning measurement methods. It only needs the airborne measurement platform to fly along a straight line and obtain the range of the measured target through two detections.

In this paper, based on the location equation of double basic path difference, a deductive method that can transform the ranging equation of double basic path difference into single basic Doppler ranging equation is presented by using the interchange relationship between frequency shift and path difference. Firstly, the linear solution of the double basic path difference positioning equation and the Doppler ranging solution with the help of difference processing are reviewed. Then, the interchange relationship between path difference and frequency shift is described. On this basis, the derivation process of transforming double path difference ranging equation into single - basis Doppler ranging equation is given.

Linear solution of one-dimensional equidistant double base linear array [4]

For the one-dimensional double base equidistant linear array shown in figure 1, the path difference between two adjacent baselines is:

$$\Delta r_{12} = r_1 - r_2 \qquad (1)$$

$$\Delta r_{23} = r_2 - r_3 \qquad (2)$$

If the midpoint of the entire array is taken as the coordinate origin, the following two geometric auxiliary equations can be listed according to the cosine theorem:

$$\begin{aligned} r_1^2 &= r_2^2 + d^2 - 2r_2 d\cos(90^0 + \theta_2) \\ &= r_2^2 + d^2 + 2r_2 d\sin\theta_2 \end{aligned} \qquad (3)$$

$$\begin{aligned} r_3^2 &= r_2^2 + d^2 - 2r_2 d\cos(90^0 - \theta_2) \\ &= r_2^2 + d^2 + 2r_2 d\sin\theta_2 \end{aligned} \qquad (4)$$

Where: r_i is the radial distance; d the length of a single baseline; θ_2 the angle of arrival of the target at the midpoint of the bistatic array.

Because $x = r_2 \sin\theta_2$, the geometric auxiliary equation can be rewritten as:

$$r_1^2 = r_2^2 + d^2 + 2d\cdot x \qquad (5)$$

$$r_3^2 = r_2^2 + d^2 - 2d\cdot x \qquad (6)$$

Where: x is the abscissa of the rectangular coordinate system.

At this time, if the path differences (1) and (2) of two adjacent baselines are substituted into the geometric auxiliary equations (5) and (6), the following binary linear equations can be obtained after sorting out the terms:

$$2d \cdot x - 2\Delta r_{12} r_2 = -d^2 + \Delta r_{12}^2 \quad (7)$$

$$2d \cdot x - 2\Delta r_{23} r_2 = d^2 - \Delta r_{23}^2 \quad (8)$$

The radial distance of the target can be directly calculated from it:

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2(\Delta r_{12} - \Delta r_{23})} \quad (9)$$

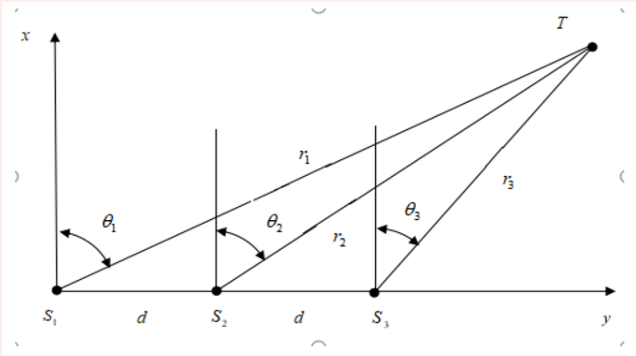

Figure 1: One dimensional double array.

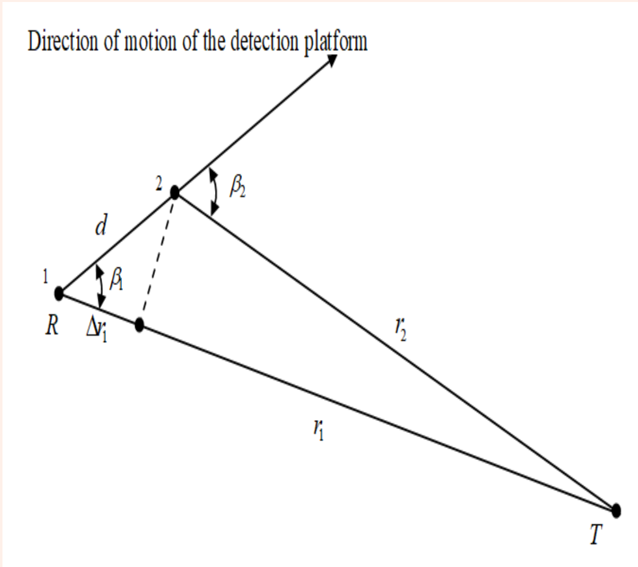

Figure 2: Doppler shift detection of moving single station.

Doppler Ranging Solution Obtained by Differential Processing [2,3]

As shown in figure 2, the Doppler receiver is installed on the moving platform to detect stationary or slowly moving targets. The Doppler frequency shift received by the receiving array is:

$$\lambda f_d = v \cos \beta \quad (10)$$

Where: f_d is is the Doppler frequency shift; λ the wavelength; ν the moving speed of the moving platform; β the leading angle.

Assume that the Doppler change rate detected by the detection platform at position 1 is:

$$\frac{\partial f_{d1}}{\partial t} = \frac{v_{t1}^2}{\lambda r_1} \quad (11)$$

Where: v_{t1} is the tangential velocity of the detection platform at position 1.

After the detection platform moves the distance d at a constant speed along the straight line, the Doppler change rate can be approximately expressed by the measured value of the Doppler frequency difference Δf_d between the two detection points in the time period Δt:

$$\frac{\partial f_d}{\partial t} = \frac{\Delta f_d}{\Delta t} = \frac{f_{d2} - f_{d1}}{\Delta t} \quad (12)$$

Combining equations (11) and (12), and using the relationship between velocity vector and its components: $v_r = \lambda f_d$, and the relationship between radial velocity and Doppler frequency shift: $v\_r = \lambda f\_d$, the following ranging formula can be obtained:

$$r_1 = \frac{\left(v^2 - \lambda^2 f_{d1}^2\right) \Delta t}{\lambda \mid \Delta f_d \mid} = \frac{d\left(v^2 - \lambda^2 f_{d1}^2\right)}{v\lambda \mid \Delta f_d \mid} \quad (13)$$

Functional Relationship Between Frequency Shift and Path Difference [5]

For the radial velocity, assuming that the change of time is short, the differential of distance to time can be converted into the ratio of path difference and time difference by using the difference calculation method:

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \quad (14)$$

Where: $r(t)$ is the radial distance; Δr the path difference; Δt the time difference.

According to the relationship between radial range change rate and Doppler frequency shift:

$$\frac{\partial r(t)}{\partial t} = \lambda f_d \quad (15)$$

From the differential processing of radial velocity, the path difference expression based on Doppler frequency shift measurement can be obtained:

$$\Delta r = \lambda f_d \Delta t \quad (16)$$

Further, for a single moving station, when the moving distance of the platform is d, the time difference that constitutes the path difference is:

$$\Delta t = \frac{d}{v} \quad (17)$$

Thus, the path difference expression based on Doppler frequency shift measurement and independent of time difference measurement is obtained:

$$\Delta r = \frac{\lambda d}{v} f_d \quad (18)$$

Direct Inference

Directly substitute the virtual path difference based on frequency shift measurement into the double basic path difference ranging formula (9) to obtain:

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2(\Delta r_{12} - r_{23})} = \frac{d\left[2v^2 - \lambda^2\left(f_{d1}^2 + f_{d2}^2\right)\right]}{2\lambda v(f_{d1} - f_{d2})} \quad (19)$$

Previous studies have shown that for two adjacent Doppler ranging solutions, the range and frequency shift are symmetrical to each other [6]. Using the measured Doppler frequency shift f_d2, the ranging solution r_1 can be solved more accurately:

$$r_1 = \frac{d\left(v^2 - \lambda^2 f_{d2}^2\right)}{v\lambda \mid \Delta f_d \mid}$$

Using the measured Doppler frequency shift f_{d1}, the ranging solution r_2 can be solved more accurately:

$$r_2 = \frac{d\left(v^2 - \lambda^2 f_{d1}^2\right)}{v\lambda \mid \Delta f_d \mid}$$

If the Doppler frequency shift on the molecule of formula (19) is approximated as: $f_{d2} \approx f_{d1}$, then there is:

$$r_2 = \frac{d\left[v^2 - \lambda^2 f_{d1}^2\right]}{\lambda v(f_{d1} - f_{d2})} \quad (20)$$

Thus, the single base Doppler ranging solution is obtained.

Conclusion

Based on the interchange relationship between path difference and frequency shift, this paper presents another method to deduce the single base Doppler ranging equation. Reference [6] has made a detailed analysis of the error characteristics of the single base Doppler ranging solution.

Both the Doppler ranging solution based on difference processing and the Doppler ranging solution based on frequency shift and path difference relationship have approximate processing, so the ranging solution obtained is only applicable to shorter baselines.

The author's recent research [7] has improved this problem. The new method can expand the baseline length to more than hundreds of kilometers by introducing frequency shift recurrence equation [8] to eliminate the adverse effects of approximation processing. This undoubtedly provides technical support for the engineering application of single base Doppler ranging solution in long baseline detection array.

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