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Mini Review

Nonlinear Fokker-Planck Equations and Turbulence in General Physical Systems

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Abstract

Recent advances in turbulent transport theory in magnetized plasmas point to applicability across a wide range of physical systems, including fluids and optical media. These findings highlight the central role of nonlinear processes and motivate generalized modeling.

Introduction

Nonlinearities have often been ignored in analytical descriptions of nature but are actually usually a fundamental part in explaining observations [1-14]. Here a quite fundamental equation is the nonlinear Fokker Planck equation which for rather general conditions can be derived from the more general Master equation [1]. This derivation holds for general systems although here we will mainly approach it from plasma physics point of view [2,3]. However, we have also explored similarities with ordinary fluid dynamics in the context of instabilities driven by temperature gradients [4]. A very important aspect of the dynamics is nonlinear frequency shifts [3,5-7].

Formulation in terms of nonlinear Fokker Planck equation

We will now start from a nonlinear Fokker Planck equation which we expect will be applicable to most media [3,8].

$$\left(\frac{\partial}{\partial t}+v.\frac{\partial}{\partial r}\right)f(r,r')=\frac{\partial}{\partial v}\left[\beta v+D^{v}\frac{\partial}{\partial v}\right]f(r,r')+S \qquad (1a)$$

where

$$\beta=\sum_j \beta_j\left|\phi_j\right|^2, \qquad (1b)$$

$$D^{v}=\sum_j d_j\left|\phi_j\right|^2, \qquad (1c)$$

The nonlinear aspect is that we consider nonlinear friction and diffusion in velocity space. We have here included only phase independent parts assuming a random phase situation. This was also done by Dupree [2]. However, if we go into the coherent limit [9,10] the results remain [11] so we can regard the Mattor Parker system [9] as the coherent limit of the Dupree result referred to as resonance broadening. As found in [3] in a stationary turbulent case where $\beta$and $D^{v}$are constants as is the usual case for plasmas, resonance broadening leads to a situation (after a time of $1/\beta$) where there is no more energy transfer between waves and resonant particles.

For fusion plasmas the source S in (1) averages to zero for low frequency perturbations responsible for transport while it has to be retained for higher frequency perturbations associated with heating [8].

Conclusion

Thus dissipative linear wave-particle resonances vanish. The physics explanation of this, as given in [14] is that an action of particles on waves has to be accompanied by an action of waves on particles. This leads to nonlinear frequency shifts [5] which change the phase velocity of waves out of resonance with particles. This physics explanation has to be valid also for waves in systems other than plasmas. Thus, we claim that our excellent agreement between theory and experiment for magnetized plasmas should be valid also for many other media [13,14].

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