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Research Article

Reduced Variables and Geometric Parameters for the Design of Thermoelectric Modules

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Abstract

In this work, the dimensioning of a thermoelectric generator is carried out, applying the calculation scheme of reduced variables, which are the reduced current density (U), the reduced efficiency (η). The key quantity to calculate the parameters geometric, cross-sectional area A and length l of the legs, is the product uk; where u is the relative current density and k the thermal conductivity. Subsequently, the average thermoelectric properties were calculated, that is: coefficient average Seebeck ($\bar{\alpha}$), average thermal conductivity ($\bar{\kappa}$) and average electrical resistivity ($\bar{\rho}$), because in reality these properties depend on temperature. Average quantities and dimensional parameters were used to calculate and analyze generator power. The influence of the ceramic plate and metal bridge is also included in this analysis, taking into account their thickness ($l_{ceramic}$, l_{metal}), electrical resistance ($R_{ceramic}$, R_{metal}) and thermal conductance ($\kappa_{ceramic}$).

Introduction

One of the most essential tasks for the development of a thermoelectric device is the design of its components [1]; to fulfill this purpose, there are various methodologies that have emerged within the fields of materials science and numerical simulation [2]. Up to now the rule that prevails for cost and ease of manufacturing is that all thermocouples of a thermoelectric module are all manufactured with the same area and length. However with the advances in 3D printing techniques with semiconductor materials and the discovery of new alloys [3,4]; The possibility of designing and manufacturing thermocouples with legs formed different length and cross-sectional area has been opened. Taking these facts into account, in this paper we show a technique for the design of thermocouples with new dimensional characteristics, the method is somewhat simple and of an appropriate scope to obtain new designs; the methodology takes advantage of the analysis with reduced variables and it is an idea that emerges from thermodynamics and consists in the separation of intensive variables from those that are extensive5. Snyder and collaborators [5,6] have developed a well-founded formulation on thermodynamic and materials science arguments, which contains all the tricks to apply the reduced variables methodology in the design of new thermocouples.
The reduced current density U is defined:

$$U = \frac{J}{\kappa \nabla T} \quad (1)$$

$$J = \frac{I}{A} \quad (2)$$

and reduced efficiency:

$$\eta_r = \frac{1 - u\frac{\alpha}{z}}{1 + \frac{1}{u\alpha T}} \quad (3)$$

in the following sections these amounts are essential for calculations.

Methods

Geometric Parameters (A, l)

The main dimensioning parameters for the design of a thermocouple are the cross- sectional area A and the length l of the legs. Applying the condition (l_p=l_n), to the equation 2:

$$I = J_p A_p = J_n A_n \quad (4)$$

in a temperature interval the current density is given by the following relation

$$Jl = \int_{T_c}^{T_h} \kappa u dT \quad (5)$$

where: κ is the thermal conductance, u is the reduced current density, combining the equations (4) y (5):

$$\frac{A_p}{A_n} = \frac{-J_n}{J_p} = \frac{-\int_{T_c}^{T_h} u_n \kappa_n dT}{\int_{T_c}^{T_h} u_p \kappa_p dT} \qquad (6)$$

the first step is then to calculate the integrals that appear in this quotient, for this purpose we have the values of the quantity uk, the thermoelectric material selected was Bi_2Te_3, see Table 1 :

Table 1: Numerical data of product uk for the material Bi2Te3.

T (K)	$u_p k_p dT$ (A/cm)	$u_n k_n dT$ (A/cm)
$T_0 = 298$	$u_p k_p(T_0) = 0.8132$	$u_n k_n(T_0) = -0.4966$
$T_1 = 323$	$u_p k_p(T_1) = 0.8350$	$u_n k_n(T_1) = -0.5206$
$T_2 = 348$	$u_p k_p(T_2) = 0.8424$	$u_n k_n(T_2) = -0.5679$
$T_3 = 373$	$u_p k_p(T_3) = 0.8435$	$u_n k_n(T_3) = -0.6312$
$T_4 = 398$	$u_p k_p(T_4) = 0.8466$	$u_n k_n(T_4) = -0.6933$
$T_5 = 423$	$u_p k_p(T_5) = 0.8603$	$u_n k_n(T_5) = -0.4454$

the integrals $-\int_{T_c}^{T_h} u_n \kappa_n dT \, y \int_{T_c}^{T_h} u_p \kappa_p dT$ are calculated by applying the fourth-order

$$\int_{x_0}^{x_4} f(x)\,dx \approx \frac{2h}{45}\left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)\right] \qquad (7)$$

for this case h is given as:

$$h = \frac{T_4 - T_0}{4} \qquad (8)$$

then the integral for the Bi_2Te_3 material type p or n is given as follows

$$\int_{T_0}^{T_4} u_{(p,n)} k_p(T)\,dT \approx \frac{2h}{45}\left[7u_{(p,n)}k_{(p,n)}(T_0) + 32u_{(p,n)}k_{(p,n)}(T_1) + 12u_{(p,n)}k_{(p,n)}(T_2) + 32u_{(p,n)}k_{(p,n)}(T_3) + 7u_{(p,n)}k_{(p,n)}(T_4)\right]$$

(9) & (10)

applying Eq. (10) to calculate the quotient (6) and using the numerical values in Table 1 , we obtain:

$$\frac{A_p}{A_n} = \frac{-(-57.77966666666667')}{83.82155555555556'} = 0.689318 \qquad (11)$$

Knowing the area ratio is possible to calculate the current densities in the legs by means of the following equation,

$$J_p = \frac{U_{total-h}}{A_{total}} \frac{1 + \frac{A_n}{A_p}}{\phi_{p-h} - \phi_{n-h}} \qquad (12)$$

where the quantity $\frac{U_{total-h}}{A_{total}}$ is the heat flux per unit area and is a value given by the designer, according to the operating conditions that the thermocouple must meet, Φ_{p-h} y $\Phi_{n-h,}$ are the thermoelectric potentials of the legs at the temperature T_h. For the T_h value that has been selected the corresponding values are:

$$\frac{U_{total-h}}{A_{total}} = 20\left(W/cm^2\right)$$

$$\Phi_{ph} = 0.36846 \text{ (V)}$$
$$\Phi_{nh} = -0.48657 \text{ (V)} \qquad (13)$$

with these values it is obtained that the value of the current density in the leg type p is:

$$J_p = 57.3245mA \qquad (14)$$

to obtain the value of the current in the leg type n, the value obtained from J_p and the quotient (5) are used, with which the following value is found,

$$J_n = 39.5148mA \qquad (15)$$

with the previously found values it is now possible to calculate the length of the legs, for which the following equation applies,

$$\int_{T_c}^{T_h} k_{n,p} u_{n,p} dT = J_{n,p} l_{n,p} \qquad (16)$$

and the following value of l is obtained,

$$l = 1.46223(mm) \qquad (17)$$

to calculate the cross-sectional areas Ap and An of the legs, we will call the total area A_{total}, for calculation purposes, we have proposed a value of $A_{total} = 1cm^2$, so we have the following system of linear equations,

$$A_p + A_n = 1$$

$$\frac{A_p}{A_n} = 0.689318 \qquad (18)$$

the results are:

$$A_p = 0.408045cm^2$$

$$A_n = 0.591955cm^2 \qquad (19)$$

with the values obtained from the geometric parameters: A_n, A_p, l_n y l_p, the dimensions of the legs have been obtained, taking into account the type of material (type p and type n) as well as the dependence of its thermoelectric properties depending on temperature, now the next step is to incorporate into the present analysis the thickness of the ceramic material plate ($l_{ceramic}$) and the thickness of the metal bridge (l_{metal}). Using these parameters, a thermocouple design like the one shown in Figure 1 is obtained.

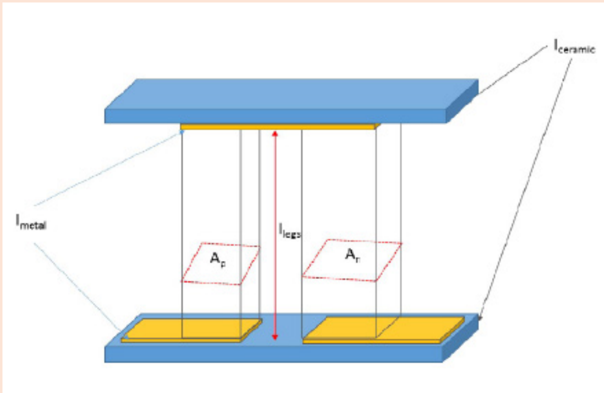

Figure 1: Design obtained from the thermocouple.

This obtained design is a realistic structure since the ceramic plate and the metal bridge are essential components of the structure of a thermocouple, and induce effects on its performance, in the following sections the average of the thermoelectric properties is obtained and these results together with the geometric parameters they are used to analyze the power of this system.

Temperature dependent properties

Table 2 contains the values of: Seebeck coefficient (α), electrical resistivity (ρ) and thermal conductivity (κ), dependent on temperature, of the Bi_2Te_3 material, n type and p type

Table 2: Numerical data of the thermoelectric properties of Bi_2Te_3.

T (K)	α_p (μV/K)	ρ_p (10^{-3}Ωcm)	κ_p (mW/cmK)
$T_0 = 298$	173	0.927	9.63
$T_1 = 323$	185	1.015	9.85
$T_2 = 348$	194	1.198	9.87
$T_3 = 373$	200	1.415	9.79
$T_4 = 398$	203	1 · 632	9.70
$T_5 = 423$	204	1 · 834	9.71
	α_n(μV/K)	**ρ_n(mW/cmK)**	**κ_n(mW/cmK)**
	-209	2.38	8
	-213	2.61	8.23
	-210	2.79	8.72
	-201	2.90	9.8
	-187	2.94	10.92
	-171	2.92	12.07

Averaged propertie

$$\bar{A} = \frac{\int_{T_c}^{T_h} A dT}{T_h - T_c} \qquad (20)$$

Using the data in Table 2 the average properties are calculated using the definition of average integral, for a quantity A,

the averages obtained are shown in the Table 3,

Table 3: Averaged thermoelectric properties of Bi_2Te_3.

Bi_2Te_3(p type)	Bi_2Te_3(n type)
$\bar{\alpha}_p$ (μV/K) = 0.000192	$\bar{\alpha}_n$ (μV/K) = −0.000206
$\bar{\rho}_p$ (10^{-3}Ωcm) = 0.00122	$\bar{\rho}_n$ (10^{-3}Ωcm) = 0.00274
$\bar{\kappa}_p$ = 0.00980 (mW/cmK)	$\bar{\kappa}_n$ = 0.00904 (mW/cmK)

with these averaged values and with the dimensional parameters obtained, an analysis of the electrical power is performed, in the following section this quantity is analyzed as a function of the thickness of the ceramic plate and the load resistance attached to the generator.

Results

Electrical Power

In this analysis we incorporate the effect of the ceramic plate and the metal bridge in thermocouple performance for which its properties: $R_{ceramic,}$ $K_{ceramic,}$ $R_{metal,}$ K_{metal} are introduced, and as geometric parameters their lengths lceramic y lmetal.

Effects of Contact on Electrical Power

So to proceed with the analysis, the well-known equations of voltage and electric current are now written in terms of the thermoelectric properties of the materials of the legs and the geometric parameters,

$$V = \frac{(\alpha_p - \alpha_n) R_{load}}{R_n + R_p + R_{met} + 2R_{cer} + R_{metn} + R_{metp} + R_{load}} (T_H - T_C) \qquad (21)$$

$$I = \frac{(\alpha_p - \alpha_n)}{R_n + R_p + R_{met} + 2R_{cer} + R_{metn} + R_{metp} + R_{load}} (T_H - T_C) \qquad (22)$$

the properties of ceramic material and metal are given in the following Table 4, it is worth mentioning that for calculation purposes a metal bridge thickness of 0.01cm has been selected,

Table 4: Thermoelectric properties of the ceramic plate and the metal bridge.

Component	Electric Resistance (R)	Thermal Conductance (κ)
Ceramic	$10(l_{ceramic})/A_{ceramic}$	$0.00000107(A_{ceramic})/l_{ceramic}$
Metal	$0{\cdot}00002(0{\cdot}01)/A_{metal}$	$0{\cdot}536(A_{metal})/0{\cdot}01$

Using the definition of electrical power:

$$P = VI \qquad (23)$$

and using Eqs. (21,22), the power P of the thermocouple is obtained,

$$P = \left(\frac{(\alpha_p - \alpha_n) R_{load}}{R_n + R_p + R_{met} + 2R_{cer} + R_{metn} + R_{metp} + R_{load}} (T_H - T_C) \right) \times \left(\frac{(\alpha_p - \alpha_n)}{R_n + R_p + R_{met} + 2R_{cer} + R_{metn} + R_{metp} + R_{load}} (T_H - T_C) \right)$$

(24) & (25)

The power graph is shown below, as a function of the thickness of the ceramic plate lc and the load resistance $R_{load,}$

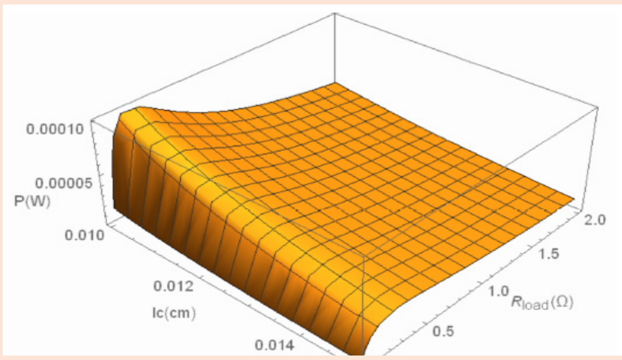

Figure 2: Graph of the power produced by the thermocouple, as a function of the thickness of the ceramic plate and the load resistance $R_{load,}$ for a value of l_{metal} = 0.01 cm.

The plot Figure 2 show that the power and efficiency increase for small values of the thickness of the ceramic material lc and that there is a specific value of R_{load} at which they reach their maximum value. For the system that has been studied in this work, the values that generate these maximum values are given in the following Table 5

Table 5: Extreme values of electrical power, achieved by the designed thermocouple in the section (II A).]

Electrical Power	$L_{c\text{-}max(cm)}$	$R_{load\text{-}max(\Omega)}$
$P_{max} = 0.0131514\ (W)$	0.15	3.01117
$P_{min} = 2.8765 * 10^{-7}\ (W)$	0.336056	0.00825062

Conclusion

The applied method has allowed obtaining a thermocouple that reaches acceptable power values, although it is still necessary to extrapolate the results to the design of a thermoelectric module; what is important in this work has been to show the main aspects of the formalism of analysis and calculation. It is remarkable that the reduced variables simplify part of the calculation work and that it is not necessary to use a great computational power to predict the behavior of the thermocouple. This model then allows selecting the specific value of load resistance (R_{load}) according to the thickness of the metal bridge and the thickness of the ceramic material that are established, and with which the thermocouple will achieve maximum power P_{max}.

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