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Review Article

Resource Extraction Evaluation Using a Mathematical Programming Framework for Surface-Underground Mining Options and Transitions Optimization

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Abstract

A mathematical programming framework based on Mixed Integer Linear Programming (MILP) model for surface-underground mining options and transition optimization for resource extraction is presented in this paper. Existing models are mainly based on a stepwise optimization approach with limited constraints which produces localized optimal solutions and are often impractical. For mineral deposits amenable to both surface and underground mining options, the MILP framework determines the most suitable mining option and associated schedule to exploit the orebody. The MILP formulation is tested and implemented on a gold deposit case study. The NPV of the optimal mining option ($ 2.515 billion) is sensitive to the gold price, ore quantity delivered from the underground mine, and delay factor associated in supporting the operational development and stopes. Positive changes in the delay factors associated with operational development support and mining stope support have more impact on the NPV than negative changes. However, the NPV is highly sensitive to the mining stope support delay than the operational development support delay.

Keywords

Mixed Integer Linear Programming; Mining Options and Transitions; Surface-Underground Mine Planning; Resource Evaluation; Ventilation Development; Geotechnical Support

Introduction

When a mineral deposit is discovered, several technical studies are rigorously conducted to ascertain the economic viability of the deposit. An important exercise is the determination of the optimum mining option required to extract the mineral. The term mining option has been used to refer to the initiatives or choices undertaken in the extractive industry to expand, change, defer, abandon, or adopt strategies for mining methods, ore extraction sequence, contracting mining services, and sometimes investment opportunities based on changing socio-economic, technological, technical, environmental or market conditions [1]. In this research, mining option refers to the extraction method essential for exploiting an orebody based on the prevailing technical, economical, safety, environmental and social conditions. The mining option could either be surface extraction methods, underground extraction methods, or both [2].

Mining options and transitions optimization studies are becoming popular in the mining industry because open pit mines are reaching their economic depth and transition needs to be considered to overcome increasing stripping ratios. Traditional methods of evaluation include manual computations with different software often not solved to optimality due to the number of permutations required. These studies could serve as essential reference sources for evaluating current discoveries of complex mineral deposits or for existing operations seeking to maximize resource recovery and revenue. Recent research works to solve the mining options and transitions problem has evolved from transition depth determination to the evaluation of the complete mining option and its associated schedule including Open Pit (OP) mining, Underground (UG) mining, simultaneous Open Pit And Underground (OPUG) mining, sequential OPUG mining, or combinations of simultaneous and sequential OPUG mining [3,4]. Reviews on the techniques for solving the mining options and transitions optimization problem have been studied and detailed by Bakhtavar [5] and Afum & Ben-Awuah [1]. The complexity of underground mining dictates that a more sophisticated optimization framework is required to solve the surface-underground mining options and transitions problem optimally. A typical approach is an optimization formulation that is fully controlled by "all" the essential constraints to obtain a global feasible solution to the surface-underground mining options and transitions problem. In a typical optimization study, there may be several competing objectives that lead to trade-offs, requiring the decision maker to choose a preferred solution among all trade-offs [6,7].

The primary objective of this research is to extend a previously formulated Mixed Integer Linear Programming (MILP) framework for surface-underground mining options and transitions planning [3] by integrating ventilation development, and geotechnical and reinforcement supports for stopes and operational developments. The objective function and constraints have been updated toward a more exhaustive and practical model for evaluating the financial benefit and resource recovery of an orebody amenable to both OP and UG mining options. The ventilation development, and rock support and reinforcement in the development openings and stopes will increase the operational costs and time (delay the mine life), and further affect the preferred mining option(s) and the quantity and sequence of rock material extracted from the stopes to the processing plants. This may impact the selected overall project strategy, including the extraction sequence and the location of the crown pillar. Thus, the surface-underground mining option(s) optimization will be greatly impacted by the addition of these constraints. In this research, to incorporate the strength of the rock formation into the formulation, rock mass classification system [8,9] such as Rock Mass Rating (RMR), Rock Structure Rating (RSR), Geological Strength Index (GSI) or Q system is used to characterize the rock formation, and then kriging [9] applied to populate the block model. The geotechnical model is incorporated into the MILP formulation as constraints that affect the cost and delays the mining operations. These classification systems could be grouped as qualitative or descriptive (e.g. GSI) and quantitative (e.g. Q system, RMR, and RSR) with RMR being more applicable to tunnels and mines. Geological and geotechnical information are fundamental to UG mining because rock mass behavior is dynamic and unique for each mine or UG opening [8,10,11]. It is therefore necessary to establish rock mass domains according to varying geological, geometrical, and stress data considerations. Knowledge of the rock mass properties (strength) and its behavior are significant for the engineering design of the various support systems for underground excavations.

For this paper, the estimated RMR values for the ore blocks and waste blocks in the case study block model were used as determinants to estimate the cost and delay in providing rock support and reinforcement in the operational development openings and stopes of the underground mine. From the RMR values, it will cost the mine more to provide support and reinforcement in the ore zones than the waste zones. Similarly, more delay may be expected during support and reinforcement works in the ore zones than in the waste zones. The estimated cost, delay, and sequence of providing rock support in the operational development openings and the stopes are then integrated as constraints in the optimization process. The time spent in installing geotechnical rock support and reinforcement are modelled as delay factors. For any operational development opening or stope, if significant time is spent to secure the area (delays are encountered), the available time required to excavate the opening or extract ore from the stope will reduce. This will not only increase the cost of operation but will also reduce the total length excavated for the operational development opening or the total tonnage of ore extracted from the stope per unit time. These new geotechnical constraints are aimed at improving the practical resource recovery for deposits that are amenable to OPUG extraction.

The next section of this paper provides a summarized literature review on surface-underground mining options and transitions optimization with notable research gaps. Section 3 details the problem definition and research approach and Section 4 introduces and explains the mathematical descriptions of the MILP model. Section 5 discusses the implementation of the model by documenting the description of the gold deposit case study, the conceptual mining strategy, rock support systems for the mine and the economic and technical data used to implement the model. Section 6 outlines the results and discussions on the implementation of the MILP framework including sensitivity analysis, while Section 7 summarizes the research conclusions and recommendations to improve the model.

Summary of Literature Review

For a typical mineral deposit, surface mining is considered highly productive, economic, and safer for workers. However, due to recent strict environmental regulations and societal expectations, Underground (UG) mine development may be preferred to shallower Open Pit (OP) operations [12]. Optimizing the extraction of a mineral deposit using both surface mining methods and UG mining methods result in the most economic decision obtained by identifying the best mining option for the deposit. Several studies have been conducted to solve the surface-underground mining transitions planning problem. Most of these published literature focuses on determining the transition depth between the open pit and underground mine, and the resulting production schedule for the mining options using simplified optimization frameworks. These models do not address the multiple objective nature of the surface-underground mining options and transitions optimization problem, and do not formulate the problem with a complete description of practical mining constraints. Specifically, the existing models do not incorporate essential developmental operations such as primary and secondary mine access, ventilation requirements, and geotechnical support and reinforcement in the optimization framework. The results from these existing models often lead to localized optimal solutions, biased solutions or solutions that lack the full range of practical constraints required for direct implementation [13-15]. Some of the current models can solve the transition problem, usually producing near optimal solutions [16,17]. Bakhtavar et al. [18] noted that, few methods (algorithms) have some disadvantages and deficiencies in finding the optimal transition depth. Finch [19] indicated that the transition problem is not thoroughly explored and therefore the results obtained may be sub-optimal. Jakubec & McCracken [20,21] stressed on the need to integrate geotechnical models in the strategic long-term mine plans at the prefeasibility stage similar to how geologic models are incorporated. Roberts, et al. [22] indicated the importance of incorporating geotechnical sequencing constraints in high level mining options studies in order to verify their impact on the optimal solution. Previous work by the authors Afum et al. [4] incorporated only primary and secondary development requirements in their MILP optimization framework.

Existing optimization algorithms used in attempting the mining options problem are Lerchs-Grossman algorithm, dynamic programming, Seymour algorithm, floating cone technique, neural network, theory of graphs, and network flows [23]. Some authors have studied the surface-underground mining options and transitions problem with available commercial software packages including Surpac Vision, Datamine's NPV Scheduler, Whittle Four-X, Geovia MineSched, integrated 3D CAD systems of Datamine, Vulcan, MineScape, MineSight, Isatis, XPAC, Mineable Reserve Optimizer (MRO), Blasor pit optimization tool, COMET cut-off grade and schedule optimizer, Datamine Studio 3, and several other widely used computer programs [13,24-26]. Most of the techniques used are not generic but scenario based, and may lead to localized optimization solutions.

Although Bakhtavar et al. [18] employed a heuristic algorithm to compare economic block values gained by both OP and UG mining on a depth flow basis to solve the surface-underground mining options and transitions problem, the results from heuristic algorithms do not offer a measure of optimality as is the case in mathematical programming optimization. Most authors who used mathematical programming to solve the mining transition problem, limit their model to the determination of transition depth, block extraction sequence and cut-off grade for the OP and UG mining operations [27-29]. Ordin & Vasil'ev [23] generated NPV curves and transition depths from OP to UG mining for Botuobinskaya pipe deposit. Similarly, other authors have developed stochastic mathematical programming models to solve the surface-underground mining options and transition problem. They focused on determination of the transition depth in 2D environment, and do not incorporate the crown pillar position as well as other essential underground mining constraints such as primary and secondary development, ventilation shaft development, and geotechnical requirements for the development openings and stopes in the optimization framework [13,30,31]. Several authors have acknowledged in their works the importance of incorporating geotechnical constraints in the OP-UG transition problem [17,22,31-33]. To verify the impact of geotechnical constraints on the optimal solution, it was recommended that such constraints need to be incorporated in subsequent studies. This however becomes difficult because optimizing UG mining operations are computationally complex and integrating it with OP mining makes it more challenging [34].

Positioning the required crown pillar in surface-underground mining operations is key to the success of such mines. Some authors pre-selected the depth of the crown pillar (transition depth) before evaluating the portions above the crown pillar for OP mining and the portions below the crown pillar for UG mining [23,29,35]. This may lead to suboptimal solutions and will require evaluating multiple crown pillar locations in a scenario-based approach. A few authors have attempted to incorporate the positioning of the crown pillar in the optimization process [3,4,15,16,30]. Their models were good improvements over previous works but were missing some essential constraints including ventilation requirements and rock strength properties required for practical implementation. Planning the transition from OP to UG mining is a complex geomechanical process which requires the consideration of rock mass properties [36,37]. Bakhtavar [38] reviewed the combined OP with UG mining methods for the past decade and noticed that the transition problem has been implemented in either simultaneous or non-simultaneous modes. He asserts that the non-simultaneous mode of combined mining is more acceptable because large-scale underground caving methods with high productivity and low costs can be used. However, in simultaneous mode, horizontal and vertical slices using underhand cut and fill with cemented backfill is more feasible to be used with OP mining. King, et al. [35] employed the biggest economic pit approach to solve the OP to UG mining planning. It handles the problem by first determining the OP mining limit before following it up with UG mining. Their approach includes preconstruction of stopes, and enforces resource capacity limit, development, extraction, and backfilling activities while excluding explicit sets of activities from completion in the same time period. It was then assumed fixed activity rates and any non-zero lower limits on the UG mine's resource constraints are removed to allow for a delayed start of the UG mine. Afum, et al. [4] implemented a mathematical programming that allows the optimization approach to decide whether the mineral deposit should be exploited with simultaneous, non-simultaneous, sequential or any of these combinations thereof. This approach ensures enough competitiveness between OP and UG mining rather than allowing OP mining to precede UG mining in the formulation.

These models in general do not include the requirements of essential underground mining infrastructure such as main access to the underground mine (shaft or decline or adit development), ventilation development, operational development (levels, ore and waste drives, crosscuts), and the required vertical development (ore passes, raises) all together. Additionally, these existing models do not incorporate the geotechnical classification of the rock formation in the surface–underground mining option and transition problem. Although these essential infrastructure and geotechnical characteristics of the rock formation are significant to underground mining operations, their added complexities make it difficult to be included in the surface-underground mining option and transition optimization models. According to Bullock [39], mine planning is an iterative process that requires looking at many options and determining which, in the long run, provide the optimum results. Using such iterative process could lead to some inferior solution(s) or sub-optimal solution(s) that do not constitute the global optimal solution.

A new Mixed Integer Linear Programming (MILP) optimization framework for evaluating the mining option(s) for a mineral deposit has been developed, implemented, and tested on a gold deposit case study. The MILP framework is based on the Competitive Economic Evaluation (CEE) approach introduced by Afum & Ben-Awuah [3]. The CEE optimization technique allows the optimizer to select the most suitable mining option(s) and extraction strategy for the deposit. The mining options evaluated are independent OP, independent UG, simultaneous OPUG, sequential OPUG, or combinations of simultaneous and sequential OPUG. Unlike in the previous MILP model by Afum, et al. [4], this new MILP framework incorporates the required UG mine ventilation development, and rock support and reinforcement of the operational development (level, ore and waste drives, crosscuts) and stopes. UG mine ventilation development often incorporates a series of bored raises and lateral drive development, and the construction of ventilation controls. Ventilation controls are a range of objects such as regulators, doors and walls which are not explicitly modeled in this paper. The term support generally refers to the various types of support used to protect underground excavations or openings and may include steel mesh, shotcrete, fibrecrete, and a variety of types of steel straps. Reinforcement on the other hand refers to the various types of rock reinforcement to help prevent rock movement and may include a variety of types of rock bolts, cable bolts, rebar, and dowel. Cable bolting in stope development can particularly be very costly and may introduce considerable time delays.

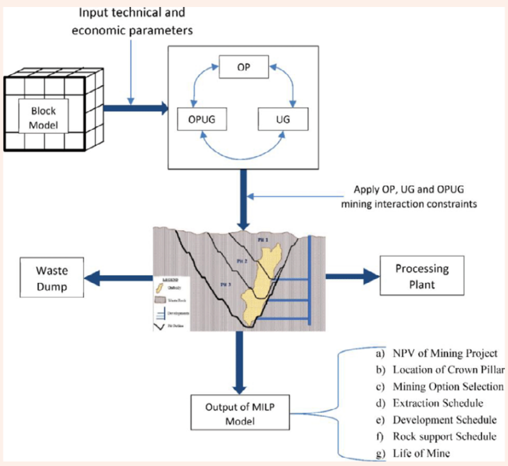

Figure 1: Schematic representation of the workflow and problem definition.

The strength of this MILP model includes the following: (a) the CEE optimization approach is unbiased; (b) the crown pillar positioning is incorporated into the optimization process and not predetermined; (c) the MILP model determines a time-dependent production schedule or extraction strategy for the selected mining option; (d) the construction of essential UG mining infrastructure (main ventilation, capital and operational development requirements) are incorporated in the formulation; (e) the type of support required to reinforce the operational development openings and stopes are considered; and (f) the MILP model evaluates all five possible mining options scenarios at the same time during optimization namely; independent Open Pit (OP) mining, independent Underground (UG) mining, simultaneous Open Pit And Underground (OPUG) mining, sequential OPUG mining, or combinations of simultaneous and sequential OPUG mining. The approach presented in Afum, et al. [4] were used as the starting point of this development.

Problem Definition and Research Approach

Figure 1 is an illustrative network of the workflow and problem definition of a deposit amenable to both OP and UG mining. A block model of a given orebody is fed into the formulated MILP framework. The necessary technical and economic parameters required to evaluate the orebody are introduced in the framework. The MILP model interrogates the orebody to determine the most suitable mining option, life of mine, ore extraction strategy and when UG mining is part of the preferred mining option. The model also determines the position of the crown pillar and the schedules for capital development (primary access, ventilation raises and accesses), secondary development (levels, drives, crosscuts), and geotechnical rock support and reinforcement delays in the operational development openings and stopes. In this research paper, an existing MILP formulation is updated with new objective function and constraints for ventilation development, and geotechnical and reinforcement supports of the stopes and operational developments [3,4]. The new MILP framework is implemented to evaluate a gold deposit case study. The MILP framework deploys the Competitive Economic Evaluation (CEE) technique in solving the mining options and transitions problem. According to Afum, et al. [4], the CEE optimization strategy is unbiased and presents a fair opportunity for each unit mining block in the block model for selection through either of the five possible mining options scenarios. As opposed to known approaches of previous authors, the MILP model evaluates all five possible mining options at the same time during optimization.

Underground development is a necessary part of underground mining as it provides the infrastructure with which production of ore can be undertaken [40]. Two types of development exist in underground mining: primary (capital) and secondary (operational) development. The main difference between these two types is life expectancy of the development. Capital development is the development of more permanent infrastructure of a mine while operational development is temporary in nature and tends to be associated with the needs of ore production (e.g., in stopes). Capital development includes shafts, declines, raises, main material handling development, and other mine accesses such as main level accesses, while operational development includes ore and waste drives, and crosscuts. Capital development such as mine ventilation and geotechnical rock mass support that could change the dynamics of the surface-underground mining options and transitions problem have been included in the MILP formulation as constraints. The multiple objective functions of the MILP model determine the following simultaneously:

a. position of the required crown pillar;
b. most suitable optimal mining option(s) for the deposit;
c. main primary access development schedule;
d. main ventilation shaft/raises schedule;
e. operational development schedule;
f. rock support and reinforcement schedule for operational development;
g. rock support and reinforcement schedule for stopes;
h. extraction schedule for the optimal mining option;
i. life of mine; and
j. Net Present Value (NPV) of the mining operation.

MILP Model for Open Pit-Underground Mining Options Planning

Assumptions and notations

The formulated MILP framework assumes that OP and UG mining has the same Selective Mining Units (SMUs). The SMUs are represented by mining blocks in general, and specifically referred to as mining-cuts in OP mining and stopes in UG mining. Stope optimization that defines the stope sizes are outside of this framework. Following after Mousavi & Sellers [41], the concept of dynamic cut-off grades in which no predefined cut-off grades are specified for each mining option is employed in this framework. The use of cut-off grade predetermines ore and waste blocks as opposed to mineralized and unmineralized blocks implemented in this paper. The location of each SMU is characterized by the coordinates of the centroid. It is assumed that a crown pillar is required for the exploitation of the ore body by underground mining. The vertical thickness of the crown pillar is assumed to be one vertical length of the unit block; thus, a single level in the block model will act as a crown pillar when UG mining becomes an option for selection in the optimization process. For UG mining, ore extraction is achieved by any of the unsupported overhand stoping methods in retreating sequence. The MILP framework is therefore limited to UG mining methods such as sublevel stoping, vertical crater retreat, and long hole stoping. Shaft position and stope definitions are assumed to be predefined before the implementation of the MILP framework. The notations for sets, indices, parameters, and decision variables used in the MILP formulation are shown in the Appendix.

Description of the MILP model

A Mixed Integer Linear Programming (MILP) model is formulated to determine the extraction of ore and waste rock materials from the open pit and/or underground mining operations, and the schedules for constructing the required main ventilation shaft, main capital development, operational development (level, ore and drives, crosscuts), and the geotechnical rock support and reinforcement requirements for the operational development and stopes. The MILP model evaluates the block model of the ore deposit and determines the best mining option(s) that maximizes the overall Net Present Value (NPV) and produces optimal strategies for reserve extraction. The MILP model builds on previous work by the integrating ventilation development, geotechnical supports and delay of the stopes and operational developments to the open pit-underground mining option(s) and transitions planning.

The MILP model is based on the Competitive Economic Evaluation (CEE) approach described by Afum, et al. [4]. The CEE optimization technique allows the optimizer to select the most suitable mining option and extraction strategy (independent OP, independent UG, simultaneous OPUG, sequential OPUG, or combinations of simultaneous and sequential OPUG) for the deposit being evaluated. During the CEE optimization process, each unit block in the block model is evaluated and the optimizer determines, if any, the blocks suitable for OP mining and/or UG mining, unmined blocks to be left in the ground as uneconomic blocks, and unmined blocks acting as the crown pillar throughout the mine life. According to Afum, et al. [4], the CEE optimization process is an unbiased approach that provides fair opportunity to each mining block for selection by a mining option.

Computing the economic and rock support parameters

$$v_j^{op,t} = \frac{1}{(1+i)^t}\left[\sum_{j=1}^{J} o_j \times g_j \times r \times (sp - sc) - \sum_{j=1}^{J} o_j \times pc\right] \quad (1)$$

$$v_p^{ug,t} = \frac{1}{(1+i)^t}\left[\sum_{p=1}^{P} o_p \times g_p \times r \times (sp - sc) - \sum_{p=1}^{P} o_p \times pc\right] \quad (2)$$

A block model of the mineral deposit with known grade, density and rock type characteristics (rock strength) is used as inputs to the MILP optimization framework. The discounted revenues obtained by selling the final commodity within each block being exploited in period t by OP mining, $v_j^{op,t}$, and UG mining, $v_p^{ug,t}$, are evaluated by Eqs. (1) and (2). The revenue from a block is a function of the available ore tonnage, grade and price of the commodity, processing recovery, selling cost of the commodity, and the extra cost of mining and processing the ore material in the block.

$$q_j^{op,t} = \frac{1}{(1+i)^t}\left[(o_j + w_j) \times cm_j^t\right] \quad (3)$$

$$q_p^{ug,t} = \frac{(o_p + w_p) \times cm_p^t}{(1+i)^t} \quad (4)$$

Computations of the discounted costs of mining all rock material within each block as waste in period t by OP mining $q_j^{op,t}$ and UG mining $q_p^{ug,t}$ are respectively given in Eqs. (3) and (4). The cost of mining a block is computed by multiplying the extraction cost per tonne of rock material (waste or ore or both) by the total rock material tonnage in the block.

$$q_{sf}^{ug,t} = \frac{pd_{sf} \times cd_{sf}^t}{(1+i)^t} \quad (5)$$

$$q_{od}^{ug,t} = \frac{od_{od} \times cd_{od}^t}{(1+i)^t} \quad (6)$$

$$q_{vd}^{ug,t} = \frac{pd_{vd} \times cd_{vd}^t}{(1+i)^t} \quad (7)$$

Similarly, the discounted costs of constructing the main UG capital (primary access) development (shaft) $q_{sf}^{ug,t}$, operational development layouts $q_{od}^{ug,t}$, and the main UG ventilation development (shaft) $q_{vd}^{ug,t}$ are respectively defined by Eqs. (5), (6) and (7). The cost of constructing the main UG access shaft is obtained by multiplying the cost per length of access shaft development by the length of capital development. The cost of constructing operational development layouts in the UG mine is obtained by multiplying the cost per length of developing a level, ore or waste drives, and crosscuts by the length of the operational development excavation. The cost of constructing the main UG ventilation shaft is obtained by multiplying the cost per length of ventilation shaft development by the length of ventilation development.

$$q_{rd}^{ug,t} = \frac{cg_{rd} \times od_{od} \times cg_{rd}^t}{(1+i)^t} \quad (8)$$

$$q_{rp}^{ug,t} = \frac{cg_{rp} \times (o_p + w_p) \times cg_{rp}^t}{(1+i)^t} \quad (9)$$

Finally, the discounted cost of providing rock support and reinforcement for the operational development openings $q_{rd}^{ug,t}$ and stopes $q_{rp}^{ug,t}$ are given in Eqs. (8) and (9). During the excavation of operational development layouts, the openings are supported and reinforced according to the strength of the rock and the proposed rock support design for such section of the opening. Providing support in the stopes are necessary to ensure the safe extraction of ore material from the stopes. The costs of providing rock support and reinforcement in the operational development openings and stopes were modelled as a function of the rock type characteristics (rock strength) and the proposed rock support design.

Objective function of the model

$$Max\sum_{t=1}^{T}\left[\begin{pmatrix}\sum_{j=1}^{J}\left(v_j^{op,t}\times x_j^{op,t}-q_j^{op,t}\times y_j^{op,t}\right)+\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_p^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right)\\ -\sum_{sf=1}^{SF}\left(q_{sf}^{ug,t}\times d_{sf}^{ug,t}\right)-\sum_{vd=1}^{VD}\left(q_{vd}^{ug,t}\times d_{vd}^{ug,t}\right)-\sum_{od=1}^{OD}\left(q_{od}^{ug,t}\times d_{od}^{ug,t}\right)\\ -\sum_{rd=1}^{RD}\left(q_{rd}^{ug,t}\times d_{rd}^{ug,t}\right)-\sum_{rp=1}^{RP}\left(q_{rp}^{ug,t}\times y_{rp}^{ug,t}\right)\end{pmatrix}\right] \quad (10)$$

The MILP model features an objective function shown in Eq. (10). The objective function maximizes the NPV of the mining project and determines the schedules for open pit ore material for processing $x_j^{op,t}$, UG mining ore material for processing $x_p^{ug,t}$, and the respective block extraction strategy for OP mining $y_j^{op,t}$ and UG mining $y_p^{ug,t}$. The objective function drives the MILP model to determine the schedule for capital development $d_{sf}^{ug,t}$, main ventilation development $d_{vd}^{ug,t}$, operational development (level, drives, crosscut) $d_{od}^{ug,t}$, and the rock support provisions for operational development layouts $d_{rd}^{ug,t}$ and underground stopes $y_{rp}^{ug,t}$. During the CEE optimization approach, if OP mining only or UG mining only is selected, the objective function of the unselected mining option and the associated decision variables become zero.

Constraints

The material extraction and geotechnical constraints of the MILP model are given in Eqs. (11) to (62). The main constraints are grouped as follows: OP mining constraints; UG mining constraints; OP and UG mining interaction constraints; crown pillar positioning constraints; main ventilation shaft (requirement) constraints; capital development constraints; operational development constraints; rock support and reinforcement provisions for operational development openings and stopes constraints; and g) non-negativity constraints.

$$T_{m,lb}^{op,t}\leq\sum_{j=1}^{J}\left[\left(o_j+w_j\right)\times y_j^{op,t}\right]\leq T_{m,ub}^{op,t} \quad (11)$$

Eq. (11) defines the mining capacity constraint for OP extraction. Open pit extraction is controlled by the continuous decision variable $y_j^{op,t}$. The total tonnage of rock material mined in each period is constrained within the acceptable lower and upper limits of the total available equipment capacity for the open pit mining operation.

$$T_{pr,lb}^{op,t}\leq\sum_{j=1}^{J}\left(o_j\times x_j^{op,t}\right)\leq T_{pr,ub}^{op,t} \quad (12)$$

Eq. (12) is the processing capacity constraint that controls the quantity of ore material being delivered from the open pit mining operation to the processing plant. The processing capacity constraint is controlled by the continuous decision variable $x_j^{op,t}$. This inequality ensures that uniform ore from the OP mining operation is fed to the processing plant throughout the mine life within acceptable lower and upper targets of the ore processing capacity of the plant in each period.

$$g_{lb}^{op,t}\leq\left[\frac{\sum_{j=1}^{J}\left(g_j\times o_j\times x_j^{op,t}\right)}{\sum_{j=1}^{J}\left(o_j\times x_j^{op,t}\right)}\right]\leq g_{ub}^{op,t} \quad (13)$$

Eq. (13) specifies the quality of ore in terms of grade content delivered from the OP mining operation to the processing plant in each period. Higher grades of ore are prioritized over lower grades of ore for mining and processing by the OP operations. This constraint does not specify the head grade of the processing plant. The minimum and maximum available ore grade in the block model are used to respectively define the lower and upper grade targets for the OP mining operation. Eq. (13) ensures that the contribution from the OP operation towards a blended processing plant head grade can be controlled.

$$x_j^{op,t}-y_j^{op,t}\leq 0 \quad (14)$$

Eq. (14) outlines the relation between the ore portion of the mining-cut and the mining-cut tonnage. The continuous variable $x_j^{op,t}$ should always be smaller than or equal to the continuous variable $y_j^{op,t}$.

$$b_j^t-\sum_{t=1}^{T}y_s^{op,t}\leq 0 \quad (15)$$

$$\sum_{t=1}^{T}y_j^{op,t}-b_j^t\leq 0 \quad (16)$$

$$b_j^t-b_j^{t+1}\leq 0 \quad (17)$$

Eqs. (15) to (17) control the vertical precedence relation of mining-cut extraction following the appropriate geotechnical mining slope for the open pit mining option. For open pit mining, nine overlying mining-cuts, $y_s^{(op,t)}$, should be extracted before the underlying mining-cut is removed.

$$\sum_{t=1}^{T}y_j^{op,t}\leq 1 \quad (18)$$

$$\left(\sum_{j=1}^{J}\frac{1}{Js}\times b_j^t\right)-y_j^t\leq 0 \quad (19)$$

$$y_j^t-y_{j-1}^t\leq 0 \quad (20)$$

Eq. (18) to (20) define the relations between OP mining block and the positioning of the crown pillar. Eq. (18) ensures that a mining-cut is extracted once in the life of the OP mine. Eq. (19) ensures that a level belongs to OP mining option when one or more mining-cuts on that level is extracted by OP mining, while Eq. (20) ensures that when a level (j) is considered for OP mining, the immediate level above it (j-1) has already been evaluated for OP mining option or left as unmined block in the crown pillar.

$$T_{m,lb}^{ug,t} \le \left[\sum_{p=1}^{P} \left[\left(o_p + w_p\right) \times y_p^{ug,t} \right] + \sum_{rp=1}^{RP} \left(cg_{rp} \times \left(o_p + w_p\right) \times y_{rp}^{ug,t} \right) \right] \le T_{m,ub}^{ug,t} \quad (21)$$

Eq. (21) is the mining capacity constraint for the UG mining operation controlled by the continuous decision variable $y_p^{ug,t}$, and the delay associated with providing support and reinforcement in the stopes $y_{rp}^{ug,t}$. $y_{rp}^{ug,t}$ is a (0,1) integer which is non-zero when stope extraction starts. This inequality ensures the total tonnage of rock material mined in each period is within acceptable lower and upper limits of the total available equipment capacity for the underground mining operation. The inequality further ensures that, delays associated with providing reinforcement in the stopes are factored into the stope production schedule. Thus, the quantity of rock material extracted in each period is controlled by the mining rate and associated geotechnical delay rate or time spent in providing rock support and reinforcement to the stopes. Delaying the supporting activity will reduce the available time required to move mineralized rock material, thus, affecting the overall material transported from the stope(s) to the processing plant.

$$T_{pr,lb}^{ug,t} \le \left[\sum_{p=1}^{P} \left(o_p \times x_p^{ug,t} \right) \right] \le T_{pr,ub}^{ug,t} \quad (22)$$

Eq. (22) is the processing capacity constraint that controls the quantity of ore delivered from the UG mining operation to the processing plant. Eq. (22) ensures the contribution of ore production from the underground mine to the overall OPUG processing capacity does not exceed a pre-defined limit. Underground ore production is also indirectly controlled by the provision of rock support and reinforcement of stopes in Eq. (21).

$$g_{lb}^{ug,t} \le \left[\frac{\sum_{p=1}^{P} \left(g_p \times o_p \times x_p^{ug,t} \right)}{\sum_{p=1}^{P} \left(o_p \times x_p^{ug,t} \right)} \right] \le g_{ub}^{ug,t} \quad (23)$$

Eq. (23) controls the quality of ore grade being delivered from UG mining. Eq. (23) ensures that rock material with high-grades can be prioritized over rock material with low-grades. This constraint does not rely on the limiting grade requirement or head grade of the processing plant but only controls the ore quality of the stope. The minimum and maximum available grade of ore in the block model are used to respectively define the lower and upper grade targets for the UG mining operation.

$$x_p^{ug,t} - y_p^{ug,t} \le 0 \quad (24)$$

Eq. (24) defines the relation between ore tonnage in the stope and the stope tonnage (both ore and waste) controlling the UG mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ should always be smaller than or equal to the continuous variable $y_p^{ug,t}$.

$$b_p^t - \sum_{t=1}^{T} y_s^{ug,t} \le 0 \quad (25)$$

$$\sum_{t=1}^{T} y_p^{ug,t} - b_p^t \le 0 \quad (26)$$

$$b_p^t - b_p^{t+1} \le 0 \quad (27)$$

Eqs. (25) to (27) control the lateral precedence relation of stope extraction on each level for the UG mining option. For UG mining, stope extraction sequence is implemented in either a retreating or advancing manner towards the main mine entrance for each underground level. There is a set of preceding stopes, $y_s^{ug,t}$, that must be removed before a stope, $y_p^{ug,t}$, is made available for removal.

$$y_p^{ug,t} - \sum_{t=1}^{T} y_{rp}^{ug,t} \le 0 \quad (28)$$

Eq. (28) defines the relations between the stoping activity and the delay associated with providing support and reinforcement in the stopes. The constraint ensures that the delays and costs of providing support in the stope is considered if that stope must be exploited. That is, the stope support variable $y_{rp}^{ug,t}$ is a (0,1) integer which becomes non-zero when a stope $y_p^{ug,t}$ is being exploited.

$$\sum_{t=1}^{T} y_{rp}^{ug,t} \le 1 \quad (29)$$

Eq. (29) ensures that a stope $y_p^{ug,t}$ is supported once in the life of the UG mine.

$$\sum_{t=1}^{T} x_p^{ug,t} \le 1 \quad (30)$$

Eq. (30) defines the reserve constraint for UG mining. This inequality ensures that each stope $x_p^{ug,t}$ on a level y_p^t is extracted once in the life of the UG mine.

$$\left(\sum_{p=1}^{P} \frac{1}{Ps} \times b_p^t \right) - y_p^t \le 0 \quad (31)$$

Eq. (31) ensures that a level belongs to UG mining option when one or more stopes on that level is extracted by UG mining.

$$b_j^t - b_p^t \le 0 \quad (32)$$

$$T_{pr,lb}^{opug,t} \le \left[\sum_{j=1}^{J} \left(o_j \times x_j^{op,t} \right) + \sum_{p=1}^{P} \left(o_p \times x_p^{ug,t} \right) \right] \le T_{pr,ub}^{opug,t} \quad (33)$$

$$y_j^t + y_c^t + y_p^t \le 1 \quad (34)$$

Eqs. (32) to (34) manage the interaction between the OP and UG mining operations. Eq. (32) represents the interaction of OP mining-cuts with UG stopes. The inequality ensures that each mining block (mining-cut/stope) is extracted by only one mining option or left as unmined block in the crown pillar or in the minefield. Eq. (33) is the combined processing capacity constraint that controls the overall mill feed. This inequality represents the contribution of ore production from both OP and UG mining options to the processing plant. Eq. (34) ensures that, a level or bench is either considered for OP mining, or UG mining, or left as crown pillar, or unmined level in the minefield. Eq. (34) further ensures that, one level or bench cannot simultaneously represent the open pit mine, underground mine and crown pillar.

$$y_c^t - y_{j-1}^t \le 0 \quad (35)$$

$$y_{c-1}^t - y_p^t \le 0 \quad (36)$$

$$y_c^t - y_c^{t+1} \le 0 \quad (37)$$

$$\sum_{c=1}^{C} y_c^t = 1 \quad (38)$$

Eqs. (35) to (38) control the positioning of the required crown pillar or transition depth and its relation to the location of the OP and UG mining operations. Eq. (35) ensures that the crown pillar is positioned immediately at the bottom of the OP mine when OP operations end while Eq. (36) ensures the crown pillar is always positioned above the level being considered for UG mining operations. Eq. (37) ensures that a level acting as the crown pillar is unmined but stays at the same location throughout the life of the mining operation while Eq. (38) ensures that one level always acts as the unmined crown pillar.

$$T_{sf,lb}^{ug,t} \le \sum_{sf=1}^{SF} \left(cd_{sf} \times d_{sf}^{ug,t} \right) \le T_{sf,ub}^{ug,t} \quad (39)$$

Eq. (39) defines the capital development capacity constraints for UG mining. This inequality ensures that the total length of capital development required in each period is within the acceptable lower and upper limits of the available equipment capacity for developing the UG mine.

$$b_{sf}^t - \sum_{t=1}^{T} d_{sf,s}^{ug,t} \le 0 \quad (40)$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} - b_{sf}^t \le 0 \quad (41)$$

$$b_{sf}^t - b_{sf}^{t+1} \le 0 \quad (42)$$

$$d_{od,s}^{ug,t} - \sum_{t=1}^{T} d_{sf}^{ug,t} \le 0 \quad (43)$$

$$\sum_{t=1}^{T} d_{sf}^{ug,t} \le 1 \quad (44)$$

Eqs. (40) to (44) control the precedence relations between the sections of capital development leading to each level and the operational development on each level. Eqs. (40) to (42) ensure that sets of capital development representing sections above a level must be completed before the capital development b_{sf}^t of that level commences. Eq. (43) ensures that development of the operational level $d_{od,s}^{ug,t}$ linking the capital development could only commence after completion of a set of required capital development $\sum_{t=1}^{T} d_{sf}^{ug,t}$ above and on that level. Eq. (44) ensures that each section of the capital development (shaft or decline) is extracted once in the life of the underground mine.

$$T_{od,lb}^{ug,t} \le \left[\sum_{od=1}^{OD} \left(od_{od} \times d_{od}^{ug,t} \right) + \sum_{rd=1}^{RD} \left(cg_{rd} \times od_{od} \times d_{rd}^{ug,t} \right) \right] \le T_{od,ub}^{ug,t} \quad (45)$$

Eq. (45) defines the operational development capacity constraint for underground mining, ensuring that the total length of operational development (level, ore and waste drives, crosscuts) required in each period is within the acceptable lower and upper limits of the available equipment capacity for developing the UG mine. The inequality further ensures that, delays associated with providing geotechnical rock support and reinforcement in the operational development are factored into the operational development schedule. Thus, the length of operational development advanced in each period is controlled by the operational development rate and geotechnical delay rate or time spent in providing rock support and reinforcement for the development openings.

$$b_{od}^t - \sum_{t=1}^{T} d_{od,s}^{ug,t} \le 0 \quad (46)$$

$$\sum_{t=1}^{T} d_{od}^{ug,t} - b_{od}^t \le 0 \quad (47)$$

$$b_{od}^t - b_{od}^{t+1} \le 0 \quad (48)$$

Eqs. (46) to (48) control the lateral precedence relations of the UG operational development required for exploiting the orebody. This ensures that depending on the mining method and support constraints, multiple and simultaneous development and stoping can occur impacting the overall stoping and development support.

$$x_p^{ug,t} - \sum_{t=1}^{T} d_{od,s}^{ug,t} \le 0 \quad (49)$$

Eq. (49) ensures that there is a set of operational development $\sum_{t=1}^{T} d_{od,s}^{ug,t}$ that must be completed before exploiting a stope $x_p^{ug,t}$ in any period.

$$\left(\sum_{od=1}^{ODs} \frac{1}{OD} \times b_{od,s}^t \right) - y_{od}^t \le 0 \quad (50)$$

In the case of Eq. (50), an underground level is activated when operational development has commenced or completed for that level.

$$y_c^t + y_{od}^t \le 1 \quad (51)$$

Eq. (51) ensures that a level selected by the optimization process as the crown pillar would not be available for operational development and vice versa.

$$d_{od}^{ug,t} - \sum_{t=1}^{T} d_{rd}^{ug,t} \le 0 \quad (52)$$

Eq. (52) constraint ensures that the geotechnical rock support and reinforcement provided in the operational development is completed immediately after completion of the operational development.

$$\sum_{t=1}^{T} d_{od}^{ug,t} \le 1 \qquad (53)$$

$$\sum_{t=1}^{T} d_{rd}^{ug,t} \le 1 \qquad (54)$$

Eqs. (53) and (54) respectively ensure that the operational development (level, ore and waste drives, crosscuts) and rock support at any section of the mine is advanced once in the life of the UG operation.

$$T_{vd,lb}^{ug,t} \le \sum_{vd=1}^{VD} \left(cd_{vd} \times d_{vd}^{ug,t} \right) \le T_{vd,ub}^{ug,t} \qquad (55)$$

Eq. (55) defines the main ventilation development capacity constraints for UG mining operations. This inequality ensures that the total length of the main ventilation development required in each period is within the acceptable lower and upper limits of the available equipment capacity for UG mine ventilation development.

$$b_{vd}^{t} - \sum_{t=1}^{T} d_{vd,s}^{ug,t} \le 0 \quad (56)$$

$$\sum_{t=1}^{T} d_{vd}^{ug,t} - b_{vd}^{t} \le 0 \quad (57)$$

$$b_{vd}^{t} - b_{vd}^{t+1} \le 0 \quad (58)$$

Eqs. (56) to (58) control the precedence relations between sections of main ventilation development leading to each level and the development on each level (drives, crosscuts). These constraints ensure that sets of main ventilation development representing sections above a level must be completed before the ventilation development b_{vd}^{t} of that level commences.

$$d_{od,s}^{ug,t} - \sum_{t=1}^{T} d_{vd,s}^{ug,t} \le 0 \qquad (59)$$

Eq. (59) ensures that development of the operational level $d_{od,s}^{ug,t}$ linking the main ventilation development could only commence after completion of a set of required main ventilation development $\sum_{t=1}^{T} d_{vd,s}^{ug,t}$ above and on that level.

$$\sum_{t=1}^{T} d_{vd}^{ug,t} \le 1 \qquad (60)$$

Eq. (60) ensures that each section of the main ventilation development is excavated once in the life of the UG mine.

$$x_j^{op,t}, y_j^{op,t}, x_p^{ug,t}, y_p^{ug,t}, d_{sf}^{ug,t}, d_{od}^{ug,t}, d_{vd}^{ug,t}, d_{rd}^{ug,t}, d_{rp}^{ug,t} \ge 0 \quad (61)$$

$$b_j^t, b_p^t, b_{sf}^t, b_{od}^t, b_{vd}^t, y_j^t, y_p^t, y_c^t, y_{od}^t \ge 0 \text{ and integers} \quad (62)$$

Eqs. (61) and (62) ensure that the decision variables for OP and UG mining, OP and UG processing, crown pillar, OP mining benches, UG mining levels, UG operational development, UG capital development, UG ventilation development, UG operational development rock support and UG stope rock support are non-negative and integers. The inequality constraints further define that the binary integer variables controlling the activities sequencing of geotechnical rock support, operational development, capital development, ventilation development, and extraction in the OP and UG mining operations are non-negative.

Implementation of the MILP Model

The Mixed Integer Linear Programming (MILP) model is implemented on a gold deposit case study. The mathematical programming framework is formulated in MATLAB 2018a [42] environment and IBM ILOG CPLEX Optimization Studio [43] was integrated into MATLAB to solve the MILP at a gap tolerance of 5%. The MILP computation was tested on an Intel(R) Xeon(R) CPU ES-1650 v4 Lenovo computer @ 3.60 GHz, with 64 GB RAM. The model is also implemented for OP mining option only by setting the ore contribution from UG mining operation to zero.

Description of the gold deposit case study

The geologic block model of the gold deposit has unit block sizes of 30 m x 30 m x 20 m. To implement the MILP model, it is assumed that the unit block sizes represent the mining-cut sizes of the OP mining operation and the stope sizes of the UG mining operation. The total resource of the ore deposit is 19.2 Mt with an average gold grade of 4.39 g/t. Figure 2 is a layout of the gold deposit showing mineralized blocks [4]. The small gold deposit shows high-grade mineralization at the top and bottom sections of the block model, while the middle sections show lower grades. The Rock Mass Rating (RMR) of the ore and waste zones in the block model indicates that the unmineralized rocks are more competent than the mineralized rocks in the block model. Table 1 is a parametric description of the gold deposit.

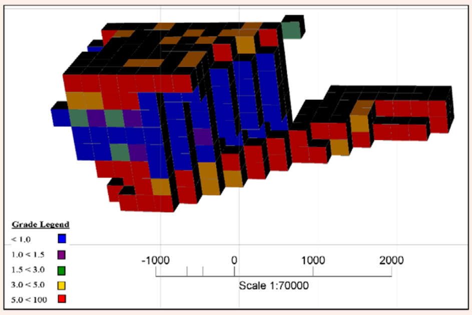

Figure 2: Layout of the gold deposit showing mineralized blocks [4].

Table 1: Parametric description of the gold deposit.

Description (units)	Value
Total mineralized material (Mt)	19.15
Minimum value of Au (g/t)	0.011
Maximum value of Au (g/t)	14.783
Average value of Au (g/t)	4.391
Variance (g/t)^2	16.539
Standard deviation (g/t)	4.067
Number of levels/benches	8
Rock mass rating of ore blocks	60
Rock mass rating of waste blocks	72

Sequencing of the underground mining operations

The MILP model is implemented for an Underground (UG) mine primarily accessed by a shaft. Shaft is used as a case study to provide independent access for UG mining in situations where UG extraction is preferred prior to transitioning to OP mining. The main ventilation is achieved by a shaft system hosting the main ventilation fans with auxiliary fans installed in the mine working areas. Lateral development including levels, waste and ore drives, and crosscuts are developed to link the mining areas for extraction of stopes. The level development extends from the main shaft to the ore and waste drives, and the crosscuts into the various stopes. Due to the strength of the rock formation, the model is implemented for an open stoping underground mining method.

The retreating sequence of stope extraction is implemented in this case study. Thus, for each level, the operational development is constructed to the end of the mine field and stope extraction begins at the end of the mine and moves towards the main entrance or primary access (shaft) of the mine. During the operational development, the openings are reinforced or supported to secure the rock mass from failing before advancing towards the end of the mine field. The rock mass reinforcement activity delays the rate of developing the operational openings in the mine. Similarly, the stopes are reinforced during the extraction process to ensure their stability. Delays associated with the provision of stope support affect the operational time of the stoping activity. Figure 3 is an isometric representation of the block model showing the unit blocks and a plan view of the underground primary access (shaft), operating development, end of minefield, ventilation shaft, and ore extraction sequence.

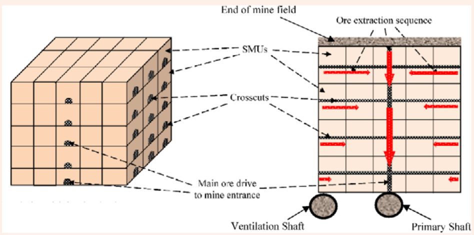

Figure 3: Isometric representation of the block model (left), and plan view of UG development layouts, mine workings and arrows showing ore extraction sequence on a level (right) modified after Afum et al. [47].

Rock support and reinforcement of the underground mining openings

A typical underground mining operation is interspersed with rock supporting and reinforcement systems for the various development openings and stopes. The rock support and reinforcement required for the development openings and operating stopes are incorporated in the implementation of the MILP model using the appropriate rock mass classification for each unit block in the block model. For the gold deposit case study, the waste material is associated with a more competent rock mass compared to the ore material. Thus, the estimated RMR values for the ore blocks is 60 while the RMR values for the waste blocks is 72. For any block that contains both ore and waste, the dominant material type is used in assigning the RMR value. With knowledge on the strength of the rock mass, support systems are designed for the operational development openings respectively for the ore rock formation and waste rock formation.

Similarly, supports are designed for each stope when it comes into operation. The cost and time (or delay) expended in providing support and reinforcement in the operational development openings and stopes were estimated based on existing mining practices and incorporated as constraints in the MILP model (Table 2). The minimum time spent in installing rock support and reinforcement per length of advancing operational development opening and per rock tonnage extracted from a stope in a day were used to determine the delay factors associated with providing geotechnical support per period for this case study. Rock support delay factors for operational development length and stopes tonnage were assumed as 0.25 and 0.1 respectively. This results from the loss of rock material movement time due to the required support works in the various locations. In general, decreasing a delay factor decreases the geotechnical support delays and hence increases the project NPV until the time spent in providing the support cannot be reduced any further. The details on the estimation of the delay factors associated with providing geotechnical rock support and reinforcement are outside the scope of this research paper.

Economic and technical data for the mining operations

The technical and economic data used for implementing the MILP model to evaluate the gold deposit was estimated from CostMine [44] and prefeasibility reports of similar gold mining operating companies in Canada [45,46]. The annual processing capacities are based on the proposed processing plant capacities for the mine while the yearly mining capacities are deduced from the ore and waste proportions of the gold deposit. Incremental bench cost of $ 2.0 per 10 m, following the NI 43-101 report of Centerra Gold Inc. and Premier Gold Mines Ltd. Sirois & Gignac [46], was used as the open pit variable cost as the depth of the pit increases. The incremental bench cost is a necessary variable cost in the reporting of mineral reserves in the mining industry and it is an important parameter in the implementation of the MILP model. It was assumed that there was no external stope dilution and mining recovery losses. Tables 2 & 3 respectively detail the technical and economic data used for the implementation of the MILP model on the gold deposit case study.

Table 2: Economic data for evaluating the gold deposit. NB: All currency is in Canadian Dollars except selling price and selling cost of gold which are in USD.

Parameter (units)	Values
Open pit mining cost ($/t)	8.0
Underground mining cost ($/t)	200.0
Processing cost ($/t)	15.0
Selling cost (USD/oz)	50.0
Selling price of gold (USD/oz)	1,400.0
Incremental bench cost ($/10 m)	2.0
Operational development cost ($/m)	7,000.0
Capital development cost ($/m)	16,000.0
Cost of supporting the operational development openings ($/m)	1,000.0
Cost of supporting the stopes ($/tonne)	80.0
Discount rate (%)	5.0
Processing recovery (%)	90.0

Table 3: Technical operational data for evaluating the gold deposit.

Parameter (units)	Values
Max open pit (OP) ore extraction capacity (Mt/year)	2.0
Min open pit (OP) ore extraction capacity (Mt/year)	0.0
Max open pit (OP) mining capacity (Mt/year)	5.0
Min open pit (OP) mining capacity (Mt/year)	0.0
Max underground (UG) ore extraction capacity (Mt/year)	1.125
Min underground (UG) ore extraction capacity (Mt/year)	0.0
Max underground (UG) mining capacity (Mt/year)	2.5
Min underground (UG) mining capacity (Mt/year)	0.0
Max processing capacity (Mt/year)	2.5
Min processing capacity (Mt/year)	0.0
Max operating development (m/year)	10,000.0
Min operating development (m/year)	0.0
Max capital (shaft) development (m/year)	40.0
Min capital (shaft) development (m/year)	0.0
Max main ventilation shaft development (m/year)	40.0
Min main ventilation shaft development (m/year)	0.0
Rock support delay factor for operational development length per year	0.25
Rock support delay factor for stopes tonnage per year	0.10

Results and Discussions

General overview

The optimized extraction option suitable for the gold deposit case study using the integrated objective Mixed Integer Linear Programming (MILP) model is a combined sequential and simultaneous Open Pit and Underground (OPUG) mining with a Net Present Value (NPV) of $ 2.515 billion. The gold mining operation commences with an independent Open Pit (OP) mine in the first 3 years. Underground (UG) mining follows simultaneously with the OP mine in the 4th and 5th years. In the 6th year, the gold project transitions sequentially from simultaneous OPUG extraction to an independent UG mining operation until the mine ends in Year 12. Figure 4 shows the ore extraction (processing) strategy and the average ore grade processed by each mining option for the gold deposit case study [47].

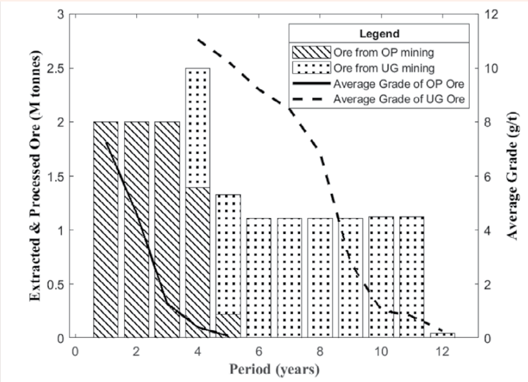

Figure 4: Ore extraction (processing) strategy and average ore grade processed by each mining option.

In Figure 4, the yearly trend of the average grade of ore extracted and processed by both mining options indicate that the optimization technique, as in all cases of mining, prioritized the relatively high-grade blocks over the lower grade blocks. The life of mine of the gold deposit case study is determined as 12 years. Ore mining recovery of 16.56 M tonnes, constituting about 86.5% of the total available mineral deposit of 19.15 M tonnes is achieved. About 7.61 M tonnes of ore is extracted by OP mining operation while 8.95 M tonnes of ore is extracted by UG mining operation. The remaining 13.5% of the mineralized rock material (about 2.59 M tonnes) is either lost in the unmined crown pillar or delivered to the waste dump as low-grade ore material. The schedule for the total rock material extracted consisting of both ore and waste rocks is shown in Figure 5. The total rock material extracted by the OP operations indicate a gradual increase in waste tonnage to maintain a uniform plant feed. In the 3rd year of mine life, there is significant waste stripping compared to ore extraction (Figures 4 & 5). Thus, there is some considerable pushback to be undertaken in the 3rd and 4th years of mine life to uncover ore material for extraction. It can be deduced that in the 4th year, when OP mining operations become unprofitable due to significant waste stripping, UG mining operation takes over as it becomes more profitable.

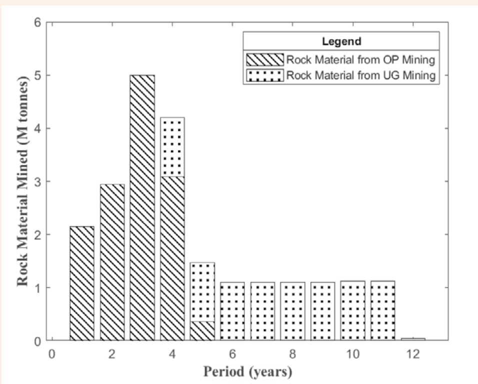

Figure 5: Mining schedule (ore and waste) for each mining option.

The schedules for the development of the primary access and the lateral secondary or operational openings (levels, waste and ore drives, crosscuts) are shown in Figure 6. The primary access development starts in year 1, during the OP mining operations and ends in year 4, before the secondary development starts. The secondary development which constitutes the operational development, occurs from the 4th year to a year before the end of mine life. Figure 7 shows the ore extraction schedule on each level of the UG mining option. The UG mining operation is localized from Levels 5 to 8. The unmined crown pillar is positioned on Level 4 and the UG ore extraction starts from the high-grade mineralized zones on Level 8 towards the low-grade mineralized zones on Level 5. The ore extraction proceeds upward from Level 8 to Level 5 through Levels 7 and 6 in that order. On each level, the stope mining is retreating from the end of the minefield towards the primary access (shaft) using open stoping mining methods.

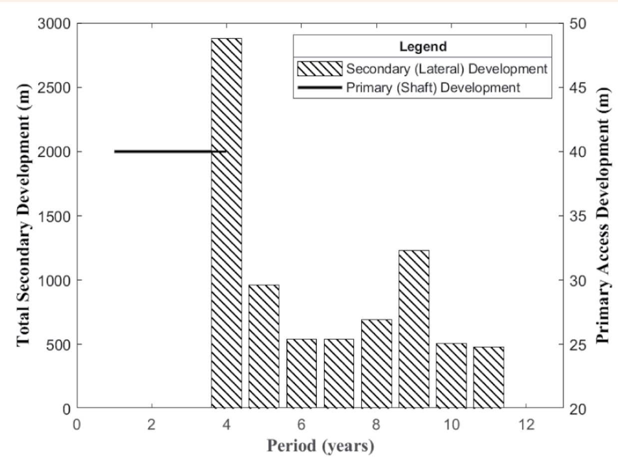

Figure 6: Primary access development and lateral secondary or operational development schedules.

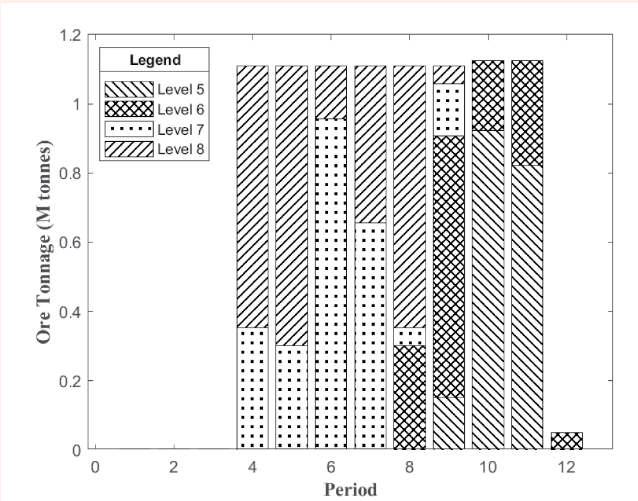

Figure 7: Ore extraction schedule on each level of the UG mining option.

The main ventilation development schedule and the delay schedules associated with providing rock support and reinforcement in the secondary or operational development openings and stopes are shown in Figure 8. As expected, development of the main ventilation shaft, a primary development, is completed before the development of the secondary openings and the associated support and reinforcement required for the operational development. The delays associated with supporting the operational development openings and stopes per period, converted to days, show that much time is required in supporting the development and open stopes to ensure smooth ore delivery.

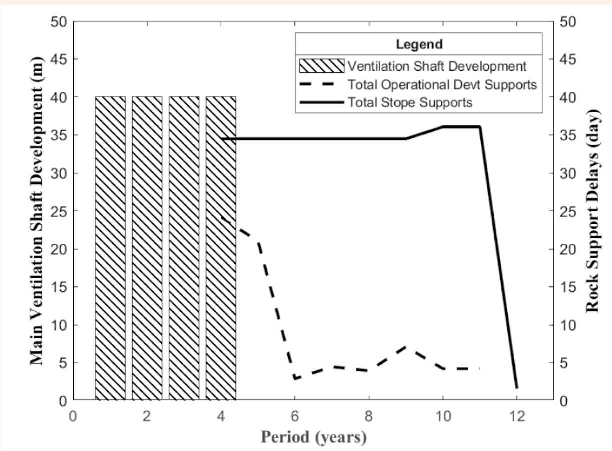

Figure 8: Schedules for main ventilation development and geotechnical rock support and reinforcement provided in the secondary development openings and stopes.

Sensitivity analysis

Due to the volatility of technical and economic parameters used as inputs in the MILP model implementation, sensitivity analysis was conducted on some selected parameters. These include: i) the delay factors for geotechnical rock support and reinforcement for operational development openings and stopes; ii) the quantity of ore being delivered from the UG mining operations to the processing plant; and iii) the price of the commodity. The model is deployed for OP mining option only when the quantity of ore being extracted and processed from the UG mine is constrained to zero. A 10.8% reduction in the gold price, from $ 1 400 to $ 1 248, changed the preferred optimal OPUG mining option with NPV of $ 2.515 billion and mineral resource recovery of 16.6 Mt (86.5%) to an independent OP mining with NPV of $ 2.232 billion and mineral resource recovery of 19.0 Mt (99.2%). Figure 9 shows the sensitivity analysis of selected parameters used in the evaluation of the gold deposit case study. The analysis shows the influence of these selected parameters on the NPV of the selected optimal mining option used as the baseline.

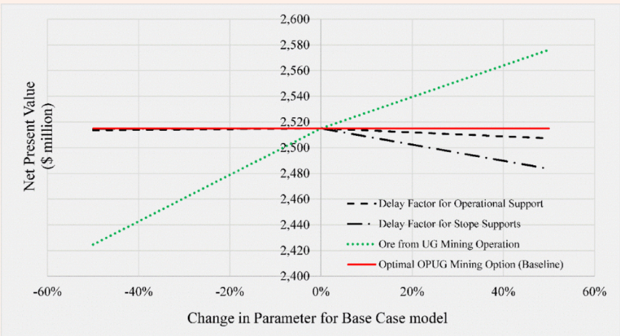

Figure 9: Sensitivity analyses for delay factors and ore processed from UG mining operation. The computed NPVs are compared to the selected optimal OPUG mining option as baseline.

From Figure 9, the NPV of the optimal mining option is significantly sensitive to the quantity of ore being delivered from the UG mining operation to the processing plant. The higher the quantity of high-grade ore being processed from UG mining operation, the likelihood UG and/or OPUG mining option become(s) favorable for selection as the optimal mining option. The sensitivity of the quantity of ore extracted by UG mining operation in an OPUG mining project was also noted by Afum, et al. [4] in their experimental work. It is further noted that, positive changes in the delay factors associated with operational development support and mining stope support have more impact on the NPV than negative changes. As expected, increasing the delay associated with the installation of support and reinforcement in the operational development openings and stopes (increasing delay factor) reduces the NPV of the selected optimal mining option. Figure 9 further shows that, there is always unavoidable minimum constant delay associated with providing some form of support to the operational development openings and stopes for the defined mining and processing targets. This does not give room for any further reduction in rock support and reinforcement delays to increase the NPV of the selected optimal UG or OPUG mining option.

Conclusions and Further Research Work

An unbiased Mixed Integer Linear Programming (MILP) model is formulated, implemented, and tested on a gold deposit case study. The mathematical programming framework is applicable for the evaluation of a mineral resource that is complex and amenable to Open Pit (OP) mining, Underground (UG) mining, simultaneous Open Pit and Underground (OPUG) mining, sequential OPUG mining, and combinations of simultaneous and sequential OPUG mining. This is also aimed at improving resource recovery for deposits that are amenable to open pit and underground extraction. In each instance of the MILP model implementation, the objective function determines the optimal mining option; the extraction strategy; the Net Present Value (NPV) of the mining operation; and the mine life. In addition, where UG mining becomes part of the preferred mining option, the position of the required crown pillar; primary access development schedule; ventilation development schedule; operational development schedule; rock support and reinforcement schedule for operational development; and rock support and reinforcement schedule for operational stopes are further determined by the objective function.

The results from the implementation indicate that the gold deposit case study is optimally exploited by a combined sequential and simultaneous Open Pit and Underground (OPUG) mining option with a crown pillar. The NPV, life of mine, location of the crown pillar, and schedules for primary access development, operational development, main ventilation shaft, and rock support and reinforcement are generated. Ore extraction starts through independent OP mining from year 1 to year 3, and after completion of the development of the main ventilation shaft, primary access (shaft) and operational openings, the mine transitions into a simultaneous OPUG mining in years 4 and 5. Ore extraction switches completely from simultaneous OPUG mining to independent UG mining from year 6 to year 12. The Net Present Value (NPV) is estimated as $ 2.52 billion at a gap tolerance of 5% and the mine life is 12 years. Sensitivity analysis on gold price indicates that, the optimal OPUG mining option with resource recovery of 86.5% will switch to an independent OP mine with NPV of $ 2.23 billion and a resource recovery of 99.2% when the gold price falls by 10.8%. The NPV is directly sensitive to the quantity of ore being delivered from the UG mining operation. It is however only sensitive to an increase in the delays associated with providing rock support and reinforcement in the secondary or operational development openings and stopes.

To improve on the versatility of the MILP framework, further research work will focus on incorporating inter-level interactions control for stope sequencing, different Selective Mining Units (SMUs) for OP and UG mining, stope backfill sequencing, and stockpile management. To control the mining strategy, it is recommended to improve the model by limiting the strip ratio to fall within specific acceptable values per the company's policy. Additional research efforts will also focus on the efficiency of the formulation and solution approach.

Appendix

Sets

$\boldsymbol{J} = \{1,\ldots,J\}$ set of all open pit mining-cuts in the model.

$\boldsymbol{J_s} = \{1,\ldots,J_s\}$ set of all open pit mining-cuts on a level in the model.

$\boldsymbol{P} = \{1,\ldots,P\}$ set of all underground stopes in the model.

$\boldsymbol{P_s} = \{1,\ldots,P_s\}$ set of all underground stopes on a level in the model.

$\boldsymbol{C} = \{1,\ldots,C\}$ set of all levels (crown pillars) in the model.

$\boldsymbol{OD_s} = \{1,\ldots,OD_s\}$ set of all underground operational development on a level in the model.

$\boldsymbol{RD} = \{1,\ldots,RD\}$ set of all underground operational development geotechnical rock support and reinforcement in the model.

$\boldsymbol{RP} = \{1,\ldots,RP\}$ set of all underground stopes geotechnical rock support and reinforcement in the model.

$\boldsymbol{SF} = \{1,\ldots,SF\}$ set of sectional underground primary development in the model.

$\boldsymbol{VD} = \{1,\ldots,VD\}$ set of sectional underground main ventilation development in the model.

$O_j(S)$ for each open pit mining-cut (j), there is a set $O_j(S) \subset \boldsymbol{J}$, defining the immediate predecessor mining-cuts that must be extracted prior to extracting mining-cut (j); where S is the total number of mining-cuts in the set $O_j(S)$.

$U_p(S)$ for each underground stope (p), there is a set $U_p(S) \subset \boldsymbol{P}$, defining the immediate predecessor stopes that must be extracted prior to extracting stope (p); where S is the total number of stopes in the set $U_p(S)$.

$C_j(S)$ for each level, there is a set $C_j(S) \subset \boldsymbol{J}$, defining the number of mining-cuts on that level that is available for open pit extraction, or left as unmined level, or crown pillar (c); where S is the total number of mining-cuts in the set $C_j(S)$.

$C_p(S)$ for each level, there is a set $C_p(S) \subset \boldsymbol{P}$, defining the number of stopes on that level that is available for underground extraction, or left as unmined level, or crown pillar (c); where S is the total number of stopes in the set $C_p(S)$.

$OD_{od}(S)$ for each level, there is a set $OD_{od}(S) \subset \boldsymbol{OD_s}$, defining the number of underground operational development on that level that must be advanced before a stope (p) is extracted; where S is the total number of operational development on the level in the set $OD_{od}(S)$.

$D_{sf}(S)$ for each level, there is a set $D_{sf}(S) \subset \boldsymbol{SF}$, defining the number of underground capital development (shaft) that must be advanced before operational development on that level can be started; where S is the total number of capital development in set $D_{sf}(S)$.

Indices

A general parameter f can take a maximum of four indices in the format of $f_{j,k}^{op,t}$. Where:

$t \in \{1,\ldots,T\}$ index for scheduling periods.

$k \in \{1,\ldots,K\}$ index for mining-blocks in the model.

$j \in \{1,\ldots,J\}$ index for open pit mining-cuts in the model.

$p \in \{1,\ldots,P\}$ index for underground stopes in the model.

$sf \in \{1,\ldots,SF\}$ index for underground primary access development (shaft) in the model.

$od \in \{1,\ldots,OD\}$ index for underground operational development in the model.

$rd \in \{1,\ldots,RD\}$ index for underground operational development geotechnical rock support and reinforcement in the model.

$rp \in \{1,\ldots,RP\}$ index for underground stopes geotechnical rock support and reinforcement in the model.

$vd \in \{1,\ldots,VD\}$ index for underground main ventilation development in the model.

$c \in \{1,\ldots,C\}$ index for crown pillars in the model.

op index for open pit mining option.

ug index for underground mining option.

$opug$ index for combined open pit and underground mining option.

Parameters

r processing recovery, the proportion of mineral content recovered.

sp^t selling price of mineral commodity in present value terms.

sc^t selling cost of mineral commodity in present value terms.

pc^t extra cost in present value terms per tonne of ore for mining and processing in period t.

$v_j^{op,t}$ the open pit (op) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of extracting mining-cut j as ore and processing it.

$v_p^{ug,t}$ the underground (ug) discounted revenue generated by selling the final product within stope p in period t minus the discounted extra cost of extracting stope p as ore and processing it.

$q_j^{op,t}$ the open pit (op) discounted cost of mining all the material in mining-cut j in period t as waste.

$q_p^{ug,t}$ the underground (ug) discounted cost of mining all the material in stope p in period t as waste.

$q_{sf}^{ug,t}$ discounted cost of constructing the main underground capital development length sf in period t.

$q_{od}^{ug,t}$ discounted cost of constructing operational development length od in period t.

$q_{vd}^{ug,t}$ discounted cost of constructing the main underground ventilation development length vd in period t.

$q_{rd}^{ug,t}$ discounted cost for underground operational development geotechnical rock support and reinforcement in period t.

$q_{rp}^{ug,t}$ discounted cost for underground stopes geotechnical rock support and reinforcement in period t.

cm_j^t cost in present value terms of extracting a tonne of rock material by open pit mining in period t.

cm_p^t cost in present value terms of extracting a tonne of rock material by underground mining in period t.

cd_{sf}^t cost in present value terms per primary access development (shaft) length sf in period t.

cd_{od}^t cost in present value terms per operational development length od in period t.

cd_{vd}^t cost in present value terms per main ventilation shaft development length vd in period t.

cg_{rd}^t cost in present value terms of providing geotechnical rock support and reinforcement per underground operational development length od in period t.

cg_{rp}^t cost in present value terms of providing geotechnical rock support and reinforcement per underground stope tonnage p in period t.

g_j estimated grade of element in ore portion of mining-cut j.

g_p estimated grade of element in ore portion of stope p.

o_j ore tonnage in mining-cut j (mineralized material).

o_p ore tonnage in stope p (mineralized material).

w_j waste tonnage in mining-cut j (non-mineralized material).

w_p waste tonnage in stope p (non-mineralized material).

pd_{sf} capital (primary access) development length (shaft) sf.

od_{od} operational development length od.

cg_{rd} underground operational development geotechnical rock support and reinforcement delay factor per rock mass classification.

cg_{rp} underground stopes geotechnical rock support and reinforcement delay factor per rock mass classification.

pd_{vd} main ventilation shaft development length vd per vertical length of a unit mining block.

$g_{lb}^{op,t}$ lower bound on acceptable average grade of element for open pit mining (op) in period t.

$g_{lb}^{ug,t}$ lower bound on acceptable average grade of element for underground mining (ug) in period t.

$g_{ub}^{op,t}$ upper bound on acceptable average grade of element for open pit mining (op) in period t.

$g_{ub}^{ug,t}$ upper bound on acceptable average grade of element for underground mining (ug) in period t.

$T_{pr,lb}^{op,t}$ lower bound on ore processing capacity requirement from open pit mining in period t.

$T_{pr,lb}^{ug,t}$ lower bound on ore processing capacity requirement from underground mining in period t.

$T_{pr,ub}^{op,t}$ upper bound on ore processing capacity requirement from open pit mining in period t.

$T_{pr,ub}^{ug,t}$ upper bound on ore processing capacity requirement from underground mining in period t.

$T_{m,lb}^{op,t}$ lower bound on available open pit mining capacity in period t.

$T_{m,lb}^{ug,t}$ lower bound on available underground mining capacity in period t.

$T_{m,ub}^{op,t}$ upper bound on available open pit mining capacity in period t.

$T_{m,ub}^{ug,t}$ upper bound on available underground mining capacity in period t.

$T_{pr,lb}^{opug,t}$ lower bound on ore processing capacity requirement from both open pit and underground mining in period t.

$T_{pr,ub}^{opug,t}$ upper bound on ore processing capacity requirement from both open pit and underground mining in period t.

$T_{sf,lb}^{ug,t}$ lower bound on capital or primary access development length for underground mining in period t.

$T_{sf,ub}^{ug,t}$ upper bound on capital development length for underground mining in period t.

$T_{od,lb}^{ug,t}$ lower bound on operational development length for underground mining in period t.

$T_{od,ub}^{ug,t}$ upper bound on operational development length for underground mining in period t.

$T_{vd,lb}^{ug,t}$ lower bound on main ventilation shaft development length for underground mining in period t.

$T_{vd,ub}^{ug,t}$ upper bound on main ventilation development length for underground mining in period t.

Decision variables

$x_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of mining-cut j to be extracted as ore and processed in period t from open pit mining.

$x_p^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope p to be extracted as ore and processed in period t from underground mining.

$y_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of mining-cut j to be mined in period t through open pit mining; fraction of y characterizes both ore and waste included in the mining-cut.

$y_p^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of stope p to be mined in period t through underground mining; fraction of y characterizes both ore and waste included in the stope.

$d_{sf}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of capital development sf to be advanced in period t for underground mining.

$d_{vd}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of main ventilation development vd to be advanced in period t for underground mining.

$d_{od}^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of operational development od to be advanced in period t for underground mining.

$d_{rd}^{ug,t} \in \{0,1\}$ continuous variable, representing the rock support and reinforcement of the operational development rd to be provided in period t for underground mining.

$y_{rp}^{ug,t} \in \{0,1\}$ binary integer variable representing the rock support and reinforcement of the stope rp to be provided in period t for underground mining.

$y_c^t \in \{0,1\}$ binary integer variable; equal to one if a level c is left as crown pillar in period t, otherwise zero.

$y_j^t \in \{0,1\}$ binary integer variable; equal to one if a mining-cut j or all mining-cuts Js on a level are extracted through open pit mining in period t, otherwise zero.

$y_p^t \in \{0,1\}$ binary integer variable; equal to one if a stope p or all stopes Ps on a level are extracted through underground mining in period t, otherwise zero.

$y_{od}^t \in \{0,1\}$ binary integer variable; equal to one if operational development od on a level is advanced in period t, otherwise zero.

$b_j^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of mining-cut for open pit mining. is equal to one if extraction of mining-cut j has started by or in period t, otherwise it is zero.

$b_p^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of stope for underground mining. is equal to one if extraction of stope p has started by or in period t, otherwise it is zero.

$b_{sf}^t \in \{0,1\}$ binary integer variable controlling the precedence of capital development for underground mining. is equal to one if capital development sf has started by or in period t, otherwise it is zero.

$b_{od}^t \in \{0,1\}$ binary integer variable controlling the precedence of operational development for underground mining. is equal to one if operational development od has started by or in period t, otherwise it is zero.

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